

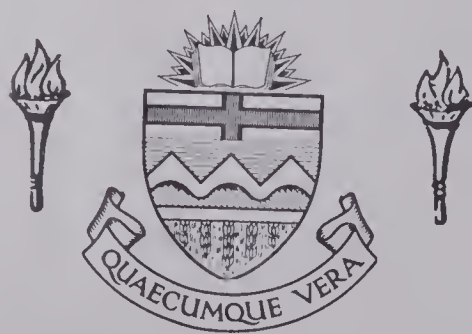
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TWO-PHASE (AIR-WATER) SLUG FLOW

IN HORIZONTAL TUBES

by



JATINDER SINGH RIAT

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES
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The undersigned hereby certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance a thesis entitled "Two-Phase (Air-Water) Slug Flow in Horizontal Tubes." submitted by Jatinder Singh Riat in partial fulfilment of the requirements for the degree of Master of Science in Chemical Engineering.

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CHAPTER I

INTRODUCTION

The term two-phase flow normally refers to the simultaneous flow of a gas or vapour and a liquid. Usually two-phase single-component flow is used to specify the simultaneous flow of a vapour and its liquid and two-phase two-component flow is used to indicate the flow of gas and a different liquid.

Engineers involved with oil and gas production have been constantly concerned with problems in two-phase flow. Predicting pressure drop through well strings, bottom hole pressure, gas required for gas lift operations and other such factors involving heat transfer, mass transfer, agitation and transportation, all require the practice of the art and science of two-phase flow. It is of some interest that the two-phase flow has at the same time, become of increasing importance in subject areas totally unrelated to the oil and gas industry. But the problems being faced are very similar to that faced by the pipeline designer.

Simultaneous two-phase flow is so complex that many simplifying assumptions have been made, which in turn reflects much disagreement among investigators. When both fluids flow simultaneously through the same pipe, the cross-sectional area for either fluid is reduced. This reduces the effective diameter of the pipe. For single-phase flow, the friction factor is inversely proportional to the fifth power of the tube diameter. Therefore, a reduction in effective diameter by one-fifth will cause the pressure drop to triple for a constant mass flow rate. But actually, the pressure drop cannot be handled by this method as two-phase flow has an additional bulk momentum transfer and an acceleration term apart from single phase momentum transfer.

Baker (6)^{*} reported that the pressure drop for two-phase flow may be ten times as much as for single-phase flow.

Recently, it is recognised that the analysis of any two-phase problem must begin with an investigation of the flow regime. Therefore, the general direction of research on two-phase flow has been toward the obtaining of solutions for specific flow geometries rather than the development of gross/overall correlations that encompass all flow regimes. In spite of this, it is very remarkable to see that such a general correlation as that of Lockhart and Martinelli (36) which has been developed without regard for flow pattern can be applied over practically the whole range of variables.

The types of flow pattern encountered in any situation depend on the fluid properties, flow rates and on the geometry of the equipment. Knowing the flow pattern and all the physical and geometric properties of the system, the pressure drop can be best predicted up to an accuracy of 25 percent (3).

Alves (2) considered the possible types of flow observed in horizontal concurrent flow of a liquid and a gas. Figure 1 shows the flow patterns as named by Alves.

1. Bubble flow
2. Plug flow
3. Stratified flow
4. Wavy flow
5. Slug flow
6. Annular flow
7. Dispersed or spray flow

* Numbers without decimal in parentheses refer to references in Bibliography.

With the exception of horizontal slug flow, most of these flow patterns have been studied quite extensively.

The slug flow can be best defined as the pattern in which either all or part of the cross-section of the tube becomes filled with liquid plugs, separated by gas-filled voids. The slug flow follows wavy flow in that a wave is picked up periodically by the more rapid moving gas. Once the wave is picked up, the gas and liquid move at the same speed. The slug flow is characterised by high pressure fluctuations when vented to atmosphere. These high fluctuations, if not damped, might cause severe damage to the receiving vessel. Therefore, it is very necessary to investigate the problem of slug flow in detail to reduce this effect or to avoid it completely, as well as to predict pressure drop.

It was not until recently in 1961 that a simplified model approximating the actual observed flow pattern for slug flow was presented by Kordyban (32). Later in 1963, Kordyban and Ranov (33) found that the model postulated by one of the authors (32) did not describe satisfactorily the behaviour of slug flow. It was said to be "based on insufficient experimental data." They (33) experimentally studied the behaviour of slug flow by measuring slug velocities, frequencies, velocity distributions, pressure drops and fluctuations and visual observation of flow mechanism by high speed motion picture camera.

In 1968, Vermeulen (44) extended Kordyban's work and described a slug flow model based on observed mechanisms of slug flow accounting for both viscous and momentum effects of the pressure drop.

In this thesis, data for horizontal two-phase slug flow have been taken for three different tube sizes to study the effect of diameter and the results compared with the correlations of Lockhart-Martinelli, Kordyban and Vermeulen. Test sections were used of sufficient length to permit full development of slugs.

CHAPTER II

LITERATURE REVIEW

The literature on two-phase flow has been reviewed quite frequently in the past fifty years by different investigators.

The literature in the field is quite extensive. A recent annotated bibliography by Kepple and Tung (30) cites over 2800 references for the period 1952 to 1962. Gouse (23) has compiled a general index of 5253 references on two-phase gas-liquid flow. Chisholm (12) reviewed literature related to the flow of gas-liquid and vapour-liquid mixtures and has referenced 308 publications from 1931 to 1962. A review by Dukler and Wicks (15) summarises the available data in tabular form and some evaluation of the various means of data analysis.

Until about 20 years ago the design of industrial equipment was based on the assumption that a mixture of phases could be treated as a homogenous fluid with the gas (or vapour) and liquid flowing at the same velocity. The phenomenon of slip was thought of in terms of vertical flow, but the slip is likely to be as great during horizontal flow (4) in many cases. The basic theory of homogeneous flow can be extended to two-phase flow if the assumption is made that the mass, momentum and energy balance apply in each phase. The incomplete knowledge of the mechanism of various interphase transfers limits the scope for general solution to the problem. The complex nature of this problem has led to the extension of single-phase flow to simplified two-phase models.

While most of the earliest work done on two-phase flow has been towards the prediction of pressure drop, recently more and more emphasis has been given to the specific flow regimes. Therefore, it is felt that the current review on the two-phase flow literature be made into following sections :

1. Pressure drop
2. Flow patterns
3. Holdup
4. Distribution of Slug Velocities

1. Pressure drop :

Many of the investigators who have measured pressure drop have suggested methods of correlation based largely on their own data. No correlation has been found to be reliable over the entire range of two-phase flow. However, these correlations could be grouped into following headings (15) :

i) Empirical Model : In this, the investigator compares the experimental pressure drop with the operating variables or the variable calculable from the single-phase flows. Usually single-phase mass velocities or pressure drops are taken as independent variables. The prediction of pressure drops from these correlations is not reliable when acceleration is significant. Some examples of this approach for horizontal flows are (11, 28, 45).

ii) Homogeneous Model : This method assumes that there is no slip between the phases and that the flow field is homogeneous. Single-phase friction factor charts are used to calculate Reynolds number based on mixture properties. Examples of correlations based on this

model are (7, 1, 8).

iii) Physical Models : A number of investigators use semi-empirical methods to calculate pressure drop for two-phase flow. This is used for particular flow patterns for developing correlation and experimental data are used to evaluate the constants. References of this approach are (36, 35).

Early work directed to a satisfactory approach to pressure measurement was carried out by Schuring (43) and Boelter and Kepner (9). The latter authors carried out work on air-water and air-oil mixtures in one-half and three-quarter inch pipes in horizontal position and 1:6 upward slope. The authors made simultaneous pressure drop measurements for horizontal and inclined flow by arranging different sections of pipe accordingly.

In 1944, Martinelli and coworkers (37) measured the pressure drop accompanying the isothermal two-phase two-component flow of air and eight various liquids, including water, for flow conditions varying from all air to all liquid in a one inch glass pipe and one-half inch galvanized iron pipe. They presented a macroscopic analysis of the various flow patterns and predicted pressure drop for certain flow conditions with two basic postulates. The first postulate is that the static pressure drop for liquid phase must equal the static pressure drop for gaseous phase, regardless of the flow details, so long as there is not appreciable pressure drop along any pipe diameter. The second postulate is that the sum of the volume occupied by the liquid and gas at any instant must equal the total volume of the pipe. The authors proposed two expressions for two-phase flow static pressure

drop based on these two postulates.

Martinelli et al (37) further assumed the two-phase pressure drop to be a function of single-phase pressure drop if either fluid were flowing alone.

In 1949, Lockhart and Martinelli (36) improved the correlations of Martinelli et al (37) using pipes varying in diameters from 0.0586 inch to 1.017 inch. Depending upon how each phase is flowing, four types of flow are shown to exist. These four regimes of flow have been correlated by means of a parameter, which is equal to the square root of the ratio of the pressure drop of the gas and liquid, assuming each phase flowing separately. Further, curves have been presented for the prediction of pressure drop and for the prediction of the fraction of the pipe filled by each phase during two-phase two-component flow in horizontal tubes.

The isothermal flow of air-water and air-oil mixtures in a 1 inch cocurrent pipe line contactor was investigated in 1954 by Alves (2). Basic data were provided on flow pattern, pressure drop and liquid holdup. The pressure drop data agreed fairly well with those predicted by Lockhart and Martinelli. The experimental values, in general, were lower than the predicted for slug and wavy flows, and higher than the predicted for annular flow.

Chenoweth and Martin (11) in 1955 presented an improved correlation for turbulent two-phase pressure drop in horizontal pipes at high pressures. This investigation was undertaken because the Lockhart and Martinelli (36) correlation was not suitable for pressures beyond 50 psig and for large diameter pipes. Many applications of two-phase

flow fell beyond these limits. The authors obtained their data for the air-water system in 1.5 and 3.0 in. horizontal pipes at near atmospheric pressure and at a pressure level near 100 psia.

Finding that the prediction of Lockhart and Martinelli was poor, especially at higher pressures, Chenoweth and Martin (11) developed their own correlation based on 264 experimental runs. The correlation represented all the data within plus or minus 50 percent and 92 percent of the data within 35 percent. They assumed that the two-phase pressure drop did depend on the flowing liquid volume fraction and pseudo single-phase pressure drop terms.

Hoogendoorn (28) in 1957 investigated the flow of air-water and air-oil mixtures in horizontal smooth pipes with inner diameter ranging from 24 mm to 140 mm and for rough pipes with an inner diameter of 50 mm. The Lockhart and Martinelli (36) correlation was reported to be valid for plug, slug and froth flow at atmospheric pressures only; for wave and mist-annular flow it was inadequate under any condition.

Hoogendoorn developed a new correlation which included influence of gas density. A standard deviation of 6 percent between his data and correlation was reported. In the higher range where most data were for slug flow type, the standard deviation was 14 percent. The standard deviation between his pressure drop data and the correlation of Lockhart and Martinelli (36) was 13 percent, while maximum deviation was plus or minus 30 percent.

In 1958, Flanigan (20) modified existing two-phase pressure drop correlations and stated that :

1. Practically all of the pressure drop in the tests occurred in the uphill section of the line.
2. The pressure drop in the line decreased as the gas flow rate increased.

The latter effect has been observed by Galegar and coworkers (22), Poettman and Carpenter (42) and Calvert (16).

The correlations for friction component of pressure drop were obtained with data in pipelines of 4, 6, 8 and 10 inch diameter taking into consideration the pipeline efficiency. The comparison of the predicted and actual pressure drop showed a maximum deviation of 13.7 percent with the majority falling within ± 5 percent.

Kordyban (32) in 1961, presented one of the first mechanistic flow models for two-phase slug flow in horizontal tubes. The author developed a correlation for the flow of a mixture of water and steam in tubes of inside diameter 0.315 inch and 0.420 inch. The most widely known and used correlation of Lockhart and Martinelli (36) was found to be practically coincident with that of Kordyban (32) for the steam-water system; although it (36) was not intended for slug flow.

Considering frictional drop only, Kordyban (32) calculated the two-phase pressure drop by dividing the liquid single-phase pressure drop by the liquid volume fraction raised to a power between 1 and 0.75. The exponent was considered an average between 1 for completely rough region and 0.75 for the smooth tubes. The correlation was developed considering that the total liquid was carried in slug and that the gaseous phase pressure drop could be neglected. The resulting

correlation greatly resembled the empirical correlation of Chenoweth and Martin (11). Kordyban concluded that a theoretical treatment of the two-phase frictional pressure drop would be better if it considered the total liquid to be carried in both slug and film.

Finding that the slug flow model presented by one of the authors (32) "did not describe satisfactorily the behaviour of slug flow, because it had been based on insufficient experimental data", Kordyban and Ranov (33), in 1963, issued a further report on slug flow in horizontal tubes. Pressure drop and pressure fluctuation were measured in a 10 ft. long 1 inch ID plexiglass tube by differential pressure transducers. Apart from this, the slug flow model was studied in detail with measurements of slug velocity and frequency to establish a modified or improved flow model.

In 1964, Dukler, Wicks and Cleveland (16) critically reviewed the existing correlations for pressure drop and holdup in horizontal flow. The total number of experimental measurements of two-phase pressure drop was found to be over 20,000. By statistically culling the data, only 2620 points were retained. Among five widely used correlations for pressure drop, the Lockhart-Martinelli (36) correlation was found to agree better with the culled data than that of Baker (6), Bankoff (7), Chenoweth and Martin (11) and Yagi (45).

Further, in 1965, Dukler (17) presented a correlation for friction pressure drop based on model theory of similarity. The correlation was checked against all of the culled data and it predicted to 10-20% where the scatter of the data itself was 5-15%.

In 1965, Faraday et al (19) studied the effect of peripheral

spray intensity and feed ratio on the two-phase pressure drop and maximum linear liquid rate for the air-water system in horizontal tubes of various diameters. The relations are characterised by systems of equations and curves for the experimental conditions employed.

Chisholm (13) in 1967 developed a theoretical basis for the Lockhart-Martinelli correlation procedure for two-phase flow in pipes. This differed from previous treatment in the method by allowing for the interfacial shear force between the phases. Some of the anomalous characteristics (e.g. of hydraulic radius) occurring in "lumped flow" were avoided. Good agreement with the Lockhart-Martinelli empirical curves were obtained. Also, for use in engineering design, some simplified equations were recommended.

2. Flow Pattern

The need for using words rather than numbers complicates the prediction of flow patterns in two-phase flow. Verbal descriptions have been necessary in the absence of quantitative measurement to characterise flow types. A number of methods has been used from time to time to present flow pattern observations and to describe the effect of external variables.

One of the earliest workers Boelter and Kepner (9) made flow pattern observations for air-water and air-oil mixtures in one-half and three-quarter inch pipes. Besides other flow patterns, slug flow was also observed.

In 1940, Cramer and Huntington (14) investigated the flow patterns of bubble, slug, froth and annular flow in a 2 inch pipe.

Martinelli et al (37) in 1944 predicted the following four flow mechanisms in two-phase flow :

1. Gas turbulent - liquid turbulent
2. Gas turbulent - liquid viscous
3. Gas viscous - liquid viscous
4. Gas viscous - liquid turbulent

The first three mechanisms were analysed macroscopically but only the first two could be experimentally noted. The fourth mechanism was stated to be very unlikely to exist in practice.

In 1949, Lockhart and Martinelli (36) confirmed the above flow mechanisms and suggested a tentative criteria for the transition between these flow mechanisms.

In the same year Kosterin (34) published his investigations on the influence of the diameter and inclination of a tube in the hydraulic resistance and flow structure of gas-liquid mixtures. Based on visual observations and photographs in 1, 2, 3 and 4 inch tubes, the following flow regions were classified :

1. Divided flow : with gas in the upper part and liquid in the lower part of the tube cross-section.
2. Quiet plug flow : with large gas bubble plugs without any froth formation.
3. Plug flow : with froth formation in the trailing part of the bubble.
4. Plug flow : with froth formation over the entire gas-liquid boundary surface.

5. Strongly dispersed flow : with gas-liquid mixture in the form of a more or less uniform froth : a gas-liquid emulsion.

The slug flow was described to occur in the region between divided flow and strongly dispersed flow. In this region gas or vapour moved in the form of large bubbles occupying considerable part of the tube diameter.

Alves (2) in 1954 gave a detailed description of general types of flow in straight horizontal pipes. The types of flow observed in horizontal cocurrent flow of a liquid plus an increasing amount of gas are (Figure 1) :

1. Bubble flow : bubbles of gas move along the upper part of the pipe at approximately the same velocity as the liquid.

2. Plug flow : alternate plugs of liquid and gas move along the upper part of the pipe.

3. Stratified flow : liquid flows along the bottom of the pipe and the gas flows above it over a smooth liquid-gas interface.

4. Wavy flow : similar to stratified flow except that the gas moves at a higher velocity and the interface is disturbed by waves travelling in the direction of flow.

5. Slug flow : a roll wave is picked up periodically by the more rapidly moving gas to form a slug which passes through the pipe at a much greater velocity than the average liquid velocity.

6. Annular flow : liquid flows in a film around the inside wall of the pipe and the gas flows at a higher velocity as a central core.

7. Dispersed or spray flow : most or nearly all of the liquid is entrained as spray by the gas.

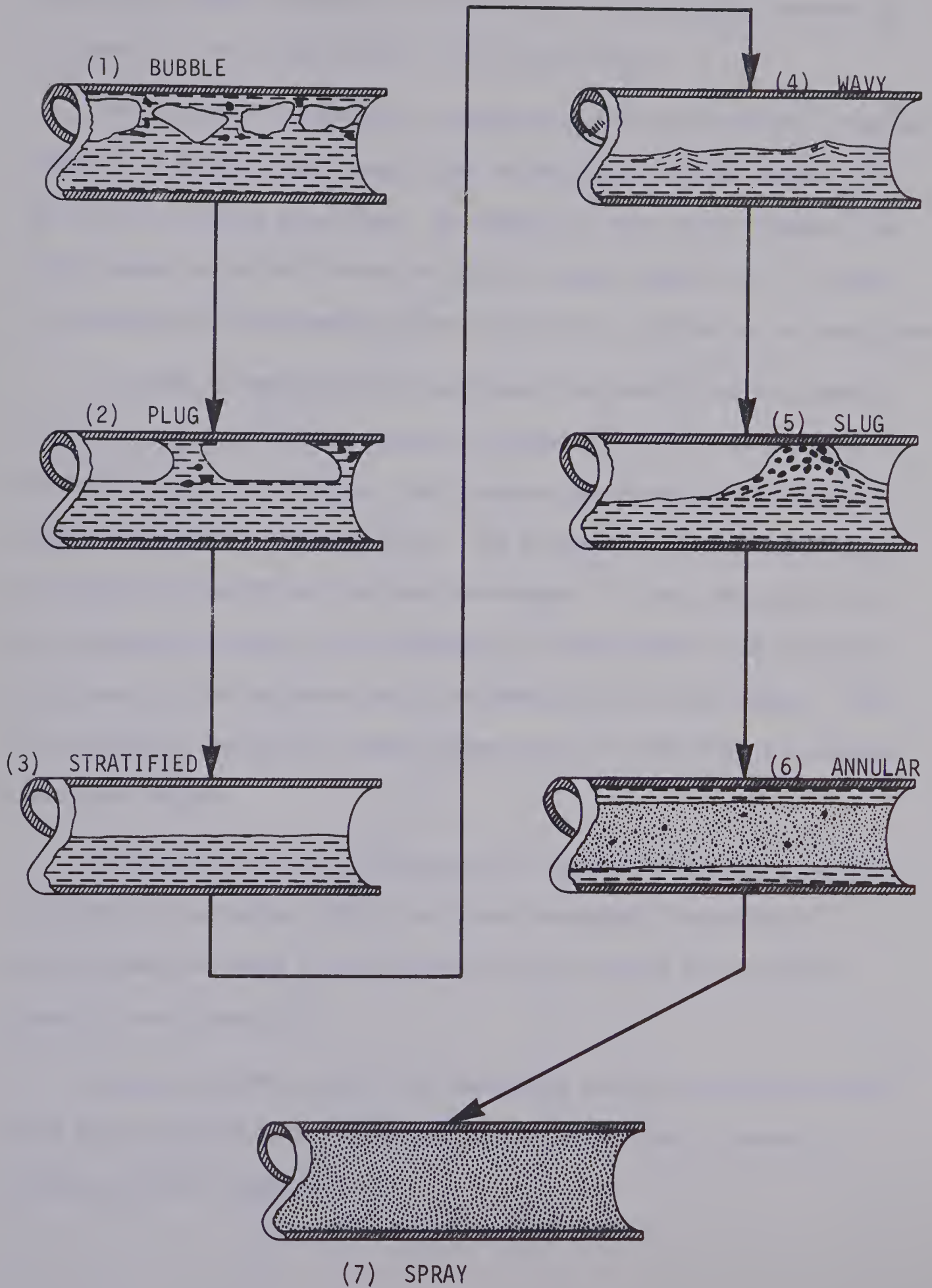


FIG. 1 - HORIZONTAL FLOW PATTERNS

Alves further emphasised that in slug flow the slugs could cause severe and/or dangerous vibrations in the equipment because of the impact of the slugs against the return bends.

The information currently available has been presented in various papers as a plot of the liquid flow or related functions against the gas flow or related functions. An example of such a plot where flow types appear as certain areas is that of Baker (Figure 2). It gives an indication of the possible flow regimes for a given set of conditions.

In 1959, Hoogendoorn (28) published the results of his investigation on the flow pattern study of air-water and air-oil mixtures in horizontal pipes. Different flow patterns were represented on the charts proposed by Kostérin (34). The relevant variables used were the mixture velocity and the gas percentage. It was concluded that for the gas-oil system, the influence of pipe diameter and viscosity being small, flow patterns could be predicted from one diagram. With the gas-water system, the same diagram could be used with an enlarged wave flow region.

Researchers in the field generally agree that to date there is no method in published literature which adequately describes or establishes just what type of flow will occur under the range of possible conditions (10).

Recently in 1969, Kern (31) presented a quick two-phase process pipe sizing method based on sizing graphs in the most frequently encountered flow regimes.

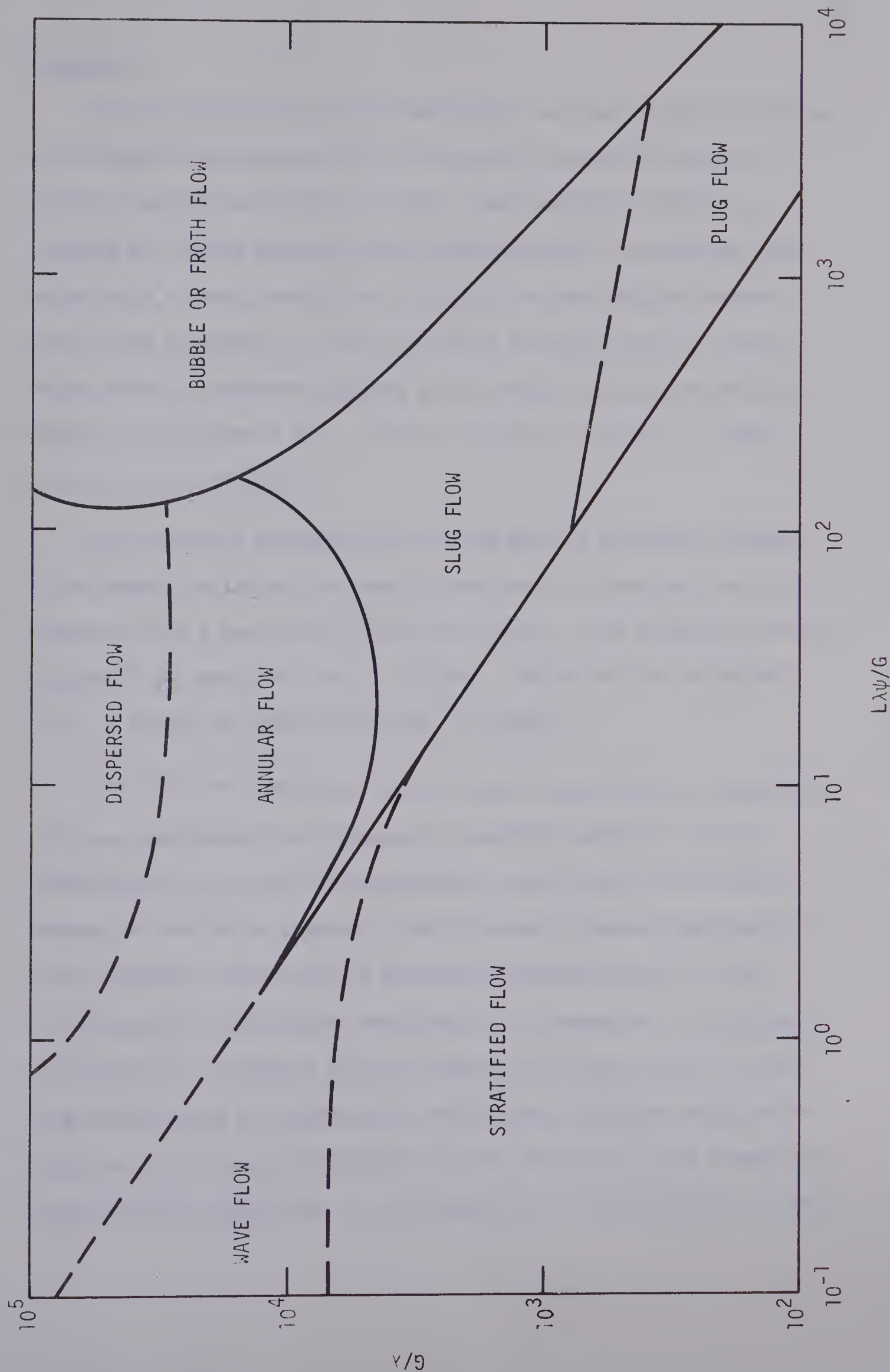


FIG. 2 - BAKER'S CHART FOR TWO-PHASE FLOW REGIONS

3. Holdup

One of the first reported measurements and correlation of holdup in two-phase two-component flow in horizontal pipes were those of Lockhart and Martinelli (36) in 1949. They measured holdup by trapping the liquid between quick closing valves. The tube was then washed with a large quantity of a volatile solvent and the excess solvent was evaporated. Curves have been presented for the liquid holdup versus a parameter defined as the square root of the ratio of liquid to gas pressure drop. For all practical purposes, a single correlation was obtained.

The report by Flanigan (20) in 1958 gives a practical example of the adverse effects of holdup in pipelines. A line was placed in operation with a gas flow of 30 MM.c.f.per day. The predicted pressure drop of 30 psi soon built up to 300 psi. The effect was accounted for by a holdup of about 200,000 gal of liquid.

In 1959, the results of liquid holdup measurements by Hoogendoorn (28) were published. He developed a capacitive method. An oil-soluble radio-active tracer homogeneously mixed with oil was used to measure the radiation intensity, which showed a linear relationship to liquid holdup. The results of Hoogendoorn showed deviations when correlated with the Lockhart-Martinelli (36) parameter. He indicated that their (36) parameter which is determined by the ratio of liquid to gas velocities was inadequate. Holdup was correlated empirically with the slip velocity between the phases. Effects of pipe diameter or liquid viscosity were found to be insignificant. The influence of gas

density was not investigated. Hoogendoorn reported a standard deviation of 0.04 (absolute value) in liquid holdup while comparing his data with the correlation.

In 1962, Petrick (41) used a gamma-ray technique to measure holdup. He used a traversing method in which the source and detector were moved across the channel to get a gas fraction profile. However, the method gave an average value rather than local values of the gas fraction. It gave no information on individual bubble frequencies and size distribution.

The same year Govier and Omer (24) reported holdup measurements employing two quick closing plug valves for the horizontal flow of air-water mixtures in 1.026 in. diameter tube. The holdup ratio increased with air velocity for all superficial water velocity reaching a maximum value of over 50. At a constant superficial air velocity the holdup ratio was reported to increase rapidly with a decrease in superficial water velocity.

Neal and Bankoff (39) in 1963 developed an improved holdup measurement method which employed a high resolution resistive probe. This probe was found capable of measuring local volumetric gas fractions, bubble frequencies and local bubble size.

The same year McManus (38) reviewed experimental techniques in two-phase flow. Several methods using contact probe, conductance element and light absorption techniques, which gave satisfactory determinations of overall density and/or void fractions were referred. No successful measurements of film thickness for non-homogeneous flows

are found.

In 1963, Kordyban and Ranov (33) developed a liquid level transducer for measuring holdup. It utilized the variation of electrical resistance with liquid level between two parallel vertical electrodes. Drainage of liquid from this transducer affected the results.

Hughmark (29) in 1965 correlated holdup in horizontal slug flow with the bubble velocity and the liquid slug Reynolds number. The holdup is represented by the ratio of the phase volumetric flow rate to the total volumetric flow rate with an experimental correlation factor in the denominator. The liquid Reynolds number is defined as

$$R_{eLs} = ((Q_L + Q_G) / A) (\rho_L / \mu_L).$$

Up to date no reliable instrument for measuring holdup is available. The above methods show correlations of holdup for measuring pressure drop.

4. Distribution of Slug Velocities

Measuring problems have made the determination of velocity distributions in two-phase flow very difficult. At very low velocity where the flow is stratified some limited data for the gas phase has been reported. But at higher rates, where the flow is dispersed to some extent, very few of the results reported have been interpreted.

The interaction between a turbulent air stream and a water film flowing parallel to it was studied by Hanaratty et al (27) in 1957. The data were correlated in terms of a liquid and a gas

Reynolds number. Thinner film or films having a lower Reynolds number were very stable. Theories presented in the literature for the initiation of waves on a liquid surface did not adequately describe the experimental results. Pressure drop measurements made simultaneously permitted the calculation of interfacial shear.

Ellis and Gay (18) in 1959 published their measurements of interfacial velocities for the air-water system in horizontal channel.

The measurements of unusual experimental significance were reported by Adorni et al (1) in 1961. Velocity and entrainment distribution were measured in pitot tube type equipment in gas phase. Data were taken for the argon-water system in a one inch vertical tube. The distribution of the two phases in core was obtained by assumption of no slip between the gas and entrained liquid.

In 1963 Kordyban and Ranov (33) reported some velocity distribution measurement for horizontal slug flow of the air-water system in a 1.25 inch plexiglass tube. Use was made of free falling trajectory of liquid emerging from the tube exit for establishing the fractions of liquid travelling at different velocities. The velocity was very high when a slug passed in comparison to that of the film. A separator gate was used to get liquid fraction at different velocity intervals.

CHAPTER III

EXPERIMENTAL EQUIPMENT

A flow sheet of the experimental apparatus is shown in Figure 3.

The test section consists of four clear lucite tubes 36.25 feet long of inside diameters $1/4$, $1/2$, 1 and $1\ 3/4$ inch. The pipelines are mounted on a hardboard fixed on four parallel aluminium ladders mounted on spindles which are fastened on a sturdy stand. The stand is located halfway along the length, for ease in altering the inclination of the tubes for future work on inclined flow.

The apparatus consists of a closed water loop, water being circulated from a 45 gallon tank with a 3 horsepower Barnes Centrifugal pump (Model 5, CCE-1) with $1\ 1/2$ inch ports. The outlet stream from the pump is split, one portion goes through a bypass or is used to saturate incoming air-stream, and the rest goes to the rotameters and magnetic flow meter. The water flow rate is measured by two Brooks rotameters (Model No. 6-1100-GTM and Model No. 8-1100) and a Foxboro magnetic flow transmitter (Type 1895-SB TS-BA, size $1/2$ inch) and Foxboro magnetic flow to current converter (Model 696). A $3/4$ inch copper tubing takes the metered water to the far end of the test section. The temperature is maintained constant in the collection tank by means of a cooling coil.

The air is supplied from the compressed air system and is vented to the atmosphere after passing through the test section. The incoming stream of air is saturated in a tank where part of the circulating water stream is sprayed from a nozzle. The saturated

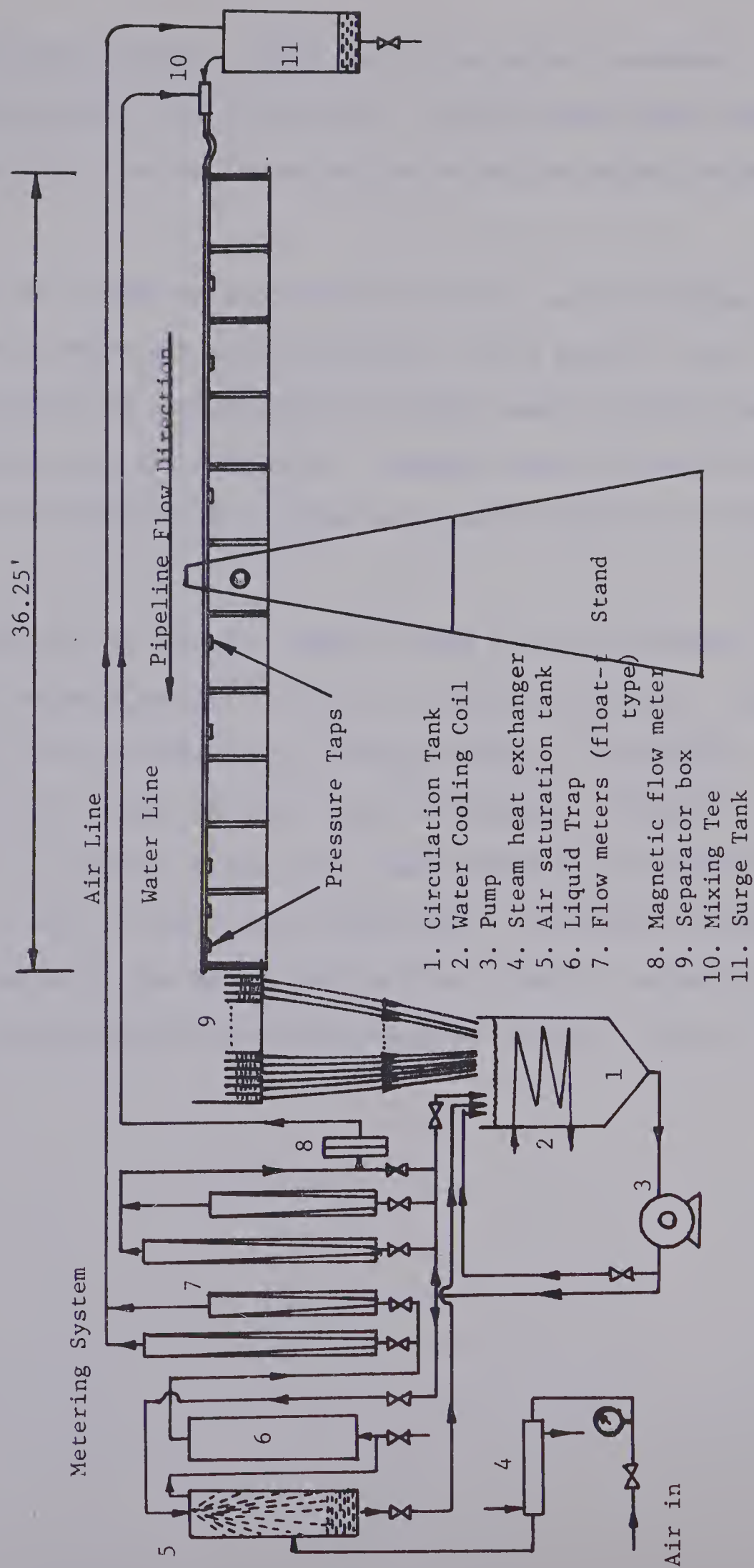


FIGURE 3 FLOW SHEET OF THE EXPERIMENTAL APPARATUS.

air then passes through a liquid trap to two Brooks rotameters (Model No. 8-1100 and Model No. 6-1100-GTM). One half inch copper tubing takes the metered air to the mixing tee at the far end of the test section.

The two fluids are introduced to the test section through a mixing tee in which air and water enter at right angles to each other. Before entering the test section the mixture passes through some tygon tubing and a 3/4 inch valve. Pressure surges and backflow of the water is controlled by a surge tank located ahead of the mixing tee.

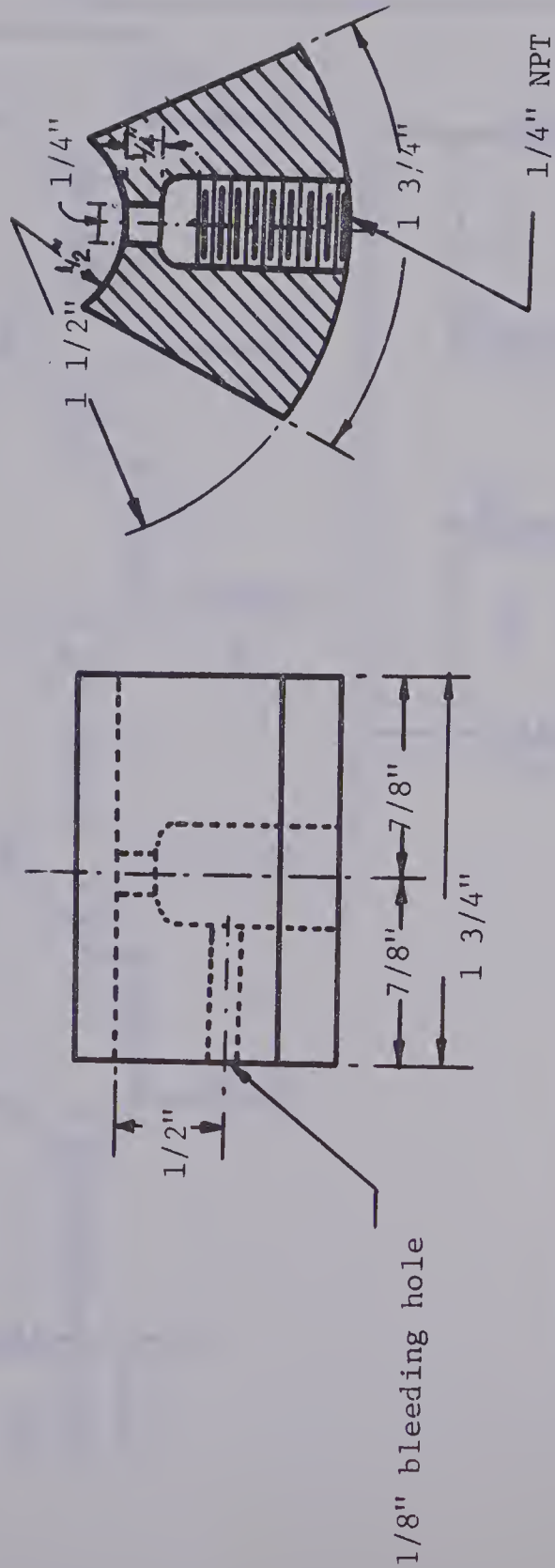
As no data on entrance length in slug flow are available, different entrance devices for the air and water are tested. Hoogendoorn (28) found that the influence of change of device is negligible beyond an entry length of sixty times the diameter. Therefore, care is taken to locate the first tap at least beyond 30, 60 and 105 inches for 1/2, 1 and 1 3/4 inch tubes respectively. Different taps are located, up to the end of the test section, along the bottom of the tube. Tap locations for different pipelines used are listed in Table 1.

TABLE I

Tube size	Tap locations from exit of the test section.
1/2 inch	4 1/2", 11 1/2", 17 3/4", 2'-9", 11'-10", 20'-1 1/2", 21'-1 1/2", 21'-5", 22'-5", 29'-2", 30'-2".
1 inch	2 3/4", 2'-0", 2'-9 1/4", 4'-5", 10'-1/2", 11'-1/2", 13'-1/2", 21'-1", 29'-1".
1 3/4 inch	2'-9 3/4", 11'-10 1/2", 13'-1/2", 14'-1", 16'-1 1/2", 21'-1", 30'-1".

A cross-sectional sketch of a pressure tap used for one inch tubing is shown in Figure 4. Pressure drop, pressure fluctuation and the presence of slug are determined by three Pace Wiancko (Whittaker Corporation Model: P7D⁺-5PSID Serial 23875 ; Model : P7D⁺-100PSID Serial 23874 and Model : P7D⁺-100PSID, Serial 23882) differential pressure transducers. These are connected to the test section at pressure taps by means of short water filled, one quarter inch polyflo tubings. Hewlett-Packard (Sanborn Model 8805A) Carrier Preamplifiers are used for the excitation and for the amplification of the signal from these transducers. The amplifier signals are recorded simultaneously on a Beckman (Type : RS Dynograph) two channel stripchart recorder.

The frequency and velocity of slugs are measured and determined by various methods. Low slug frequencies are measured visually with a stopwatch. High frequencies are sensed by a conductivity probe through a non-converting pulse amplifier and astable multivibrator and counted on a Beckman (Model : 7350R) Universal EPUT and Timer. A circuit diagram of the slug length timer is shown in Figure 5. Frequencies are



Not to scale
Material of construction: Lucite

FIGURE 4 A SKETCH OF A PRESSURE TAP FOR THE 1-INCH TUBE.

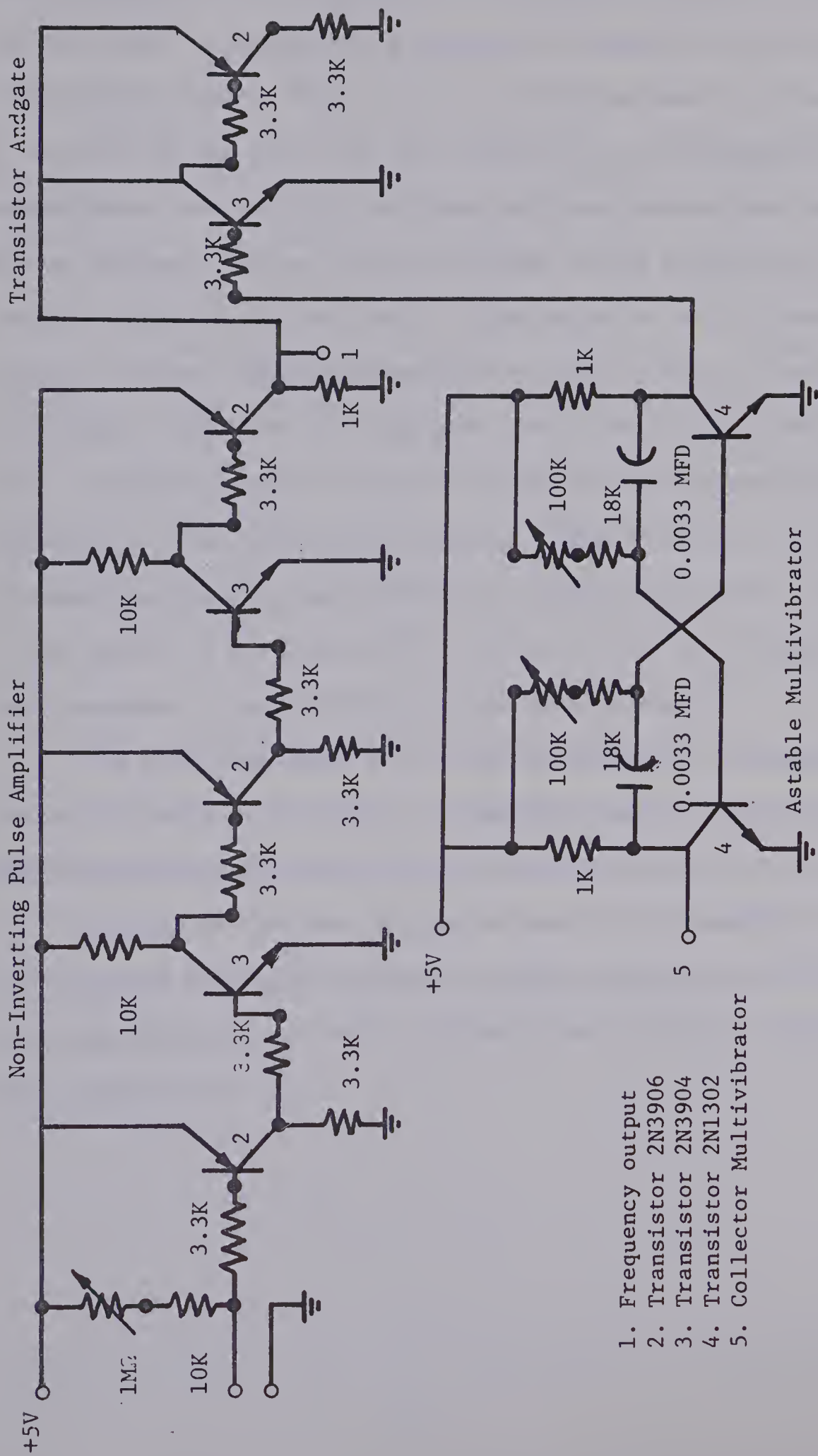


FIGURE 5. SLUG LENGTH TIMER CIRCUIT

also determined from the stripchart recording by noting the chart speed and the number of slugs over a particular distance on the chart from the pressure pulses. Slug velocities are determined from the trajectory at the exit of the tube into the separator box, consisting of 38 compartments, one and half inch wide with one-quarter inch sloped bottom openings. Screen wires are placed on the compartments to avoid too much splashing of the liquid. Slug velocity is calculated from the relation between compartment number and slug velocity by noting the compartment into which the slug goes for a particular liquid and gas rate. Volumetric rates of flow of liquid into each compartment is also measured for film flow characteristics. Slug velocity is also determined by measuring the time lag between the pressure pulses for a given slug at two points in the test section with two pitot tube probes by a strip-chart recorder. Slug velocity is also noted visually.

The effective density of slugs is determined by measuring the amount of liquid in the slugs, the percent time on slugs from conductivity probe and knowing the slug velocity and size of the tube.

The end of the test section is open to the atmosphere in order to facilitate venting of air and to collect slugs and film for determining their characteristics. Water returns to the circulation tank through the separator box.

CHAPTER IV

EXPERIMENTAL PROCEDURE

The pump is switched on to allow water to attain a steady temperature by circulation. Meanwhile, electric equipments viz., preamplifiers, oscilloscope, stripchart recorder, magnetic flow to current converter, are allowed to warm up. Then the transducer phase angle is corrected up to six degrees. This range is chosen as it has negligible effect on the calibration or operation of the inductance bridge transducers used. The transducers are then balanced and calibrated with either a mercury or a water manometer. Calibrations with water manometers are found to be necessary for the one inch and one and three-quarter inch tubes as pressure drops lie in the 0-1 psi range. Then the zeros are set for the stripchart recorder and the magnetic flow meter converter output recorder.

When steady conditions are reached the valves to the air rotameters are opened first and then magnetic flow meter or water rotameter valves are opened. The magnetic flow meter is used for one inch and one and three-quarter inch tubes as flows are limited to 3 gpm in the water rotameters and higher flow rates are needed for the distinct slug flow region. Then the water line to the air saturation tank is opened and the needle valve at the bottom of the tank is adjusted for maintaining a liquid trap. This step was not followed later on as air saturation was found to have negligible effect.

Data are taken at a constant air rate with varying water rates through the region of slug flow. This order is followed because from

Baker's chart it is seen that if the water rates are held constant and the air rate varied, several flow patterns are crossed before the slug flow pattern is reached.

The volumetric flow rates of the film into each compartment of the separator box are measured. Also the fraction of liquid in slug is measured and the compartment into which the slug goes is noted to calculate the slug velocity. Simultaneously, the slug frequency is noted from the Beckman EPUT meter and Timer. Checks of the slug velocity and frequency are made by timing the passage of slug between two pressure taps and by counting the number of slugs in a certain time interval respectively. The slug velocity is also determined by measuring the time lag between the pressure pulses for a given slug at two points in the test section from pitot tube probe with a stripchart recorder. Another method used for frequency measurement consists of counting the number of slug pressure pulses recorded on the stripchart recorder of the pressure measuring device, for a given period of time. Visual checks for slug frequency and velocity are also made.

The pressure drops and fluctuations from each of the two pressure transducers are recorded on the two-channel stripchart recorder and for each set of data the base lines are noted. To ensure that the pressure fluctuation data are correct, the pressure tap is chosen at a distance of a slug length from the tube exit.

A high-speed 16 mm motion picture camera is used to permit visual observation of the flow mechanism. Red dye is used to produce a better contrast between phases.

Normally three sets of data will suffice. As many as six sets of data are taken for slug velocity, slug frequency, pressure drop and pressure fluctuations to ensure accuracy.

CHAPTER V

RESULTS AND DISCUSSION

5.1 Flow Patterns

Different flow patterns were observed for the concurrent flow of liquid and gas for one-quarter, one-half, one, and one and three-quarter inch diameter tubes in investigating slug flow.

In the one-quarter inch tube only mist and annular flows were observed. This was due to the following two reasons. Firstly, the rotameters were too large to give flow rates necessary for slug flow in this diameter tube. Secondly, the viscosity and the surface tension are prominent in determining the flow pattern in small diameter tubes (26). This was not of interest in the study.

For the one-half inch tube, at low water rate with increasing amount of gas, stratified, wavy and annular flows were observed successively. At high water rates, stratified, plug, slug and froth flows occurred. This behaviour has been observed by Alves (2) and Vermeulen (44).

In the one-inch tube, stratified, wavy, slug and froth flows were observed. The flow rate limitations of the equipment prevented formation of annular, mist or dispersed flow.

Only plug, stratified, wavy and slug flow could be observed for the one and three-quarter inch tube because of limitations of the flow rates of the equipment.

One important visual observation noted for all the tubes during

slug flow was that the slugs started to become frothy at a no-slip velocity of about 3 ft/sec. However, the flow was still recognisable as slug flow at velocities higher than 3 ft/sec by its stability. Some authors classify this frothy slug flow as froth-flow.

5.2 Slug Flow Characteristics

Slug velocity, pressure fluctuations, pressure drop, slug density, slug frequency and fraction of liquid in slug were measured and studied by different methods. These results are presented in compact form in graphs and tables in the sections following.

5.2.1 Slug Velocity

A representative plot of no-slip or superficial slug velocity, $(Q_L + Q_G) / \pi r^2$ versus the mass rate of water at various air rates for the one-half inch diameter tube is shown in Figure 6. The table for the sets of data taken for all the tube sizes is given in Appendix A.

Figure 6 shows that the slug velocity is a stronger function of air rate than of water rate. The slope of the curves remain nearly constant with increasing air rates. This does not agree with the data of Vermeulen (44) where the slopes of the curves increase sharply with increasing air rates. This could be because his data were taken at higher air rates for the one-half inch tube. However, the trend seems to be similar with his data at lower rates of air.

The slug velocity measured by different methods agreed reasonably well and a graph of the no-slip velocity versus the measured slug velocity is shown in Figure 7 for all three tubes. The measured slug velocity values plotted represent the average of about

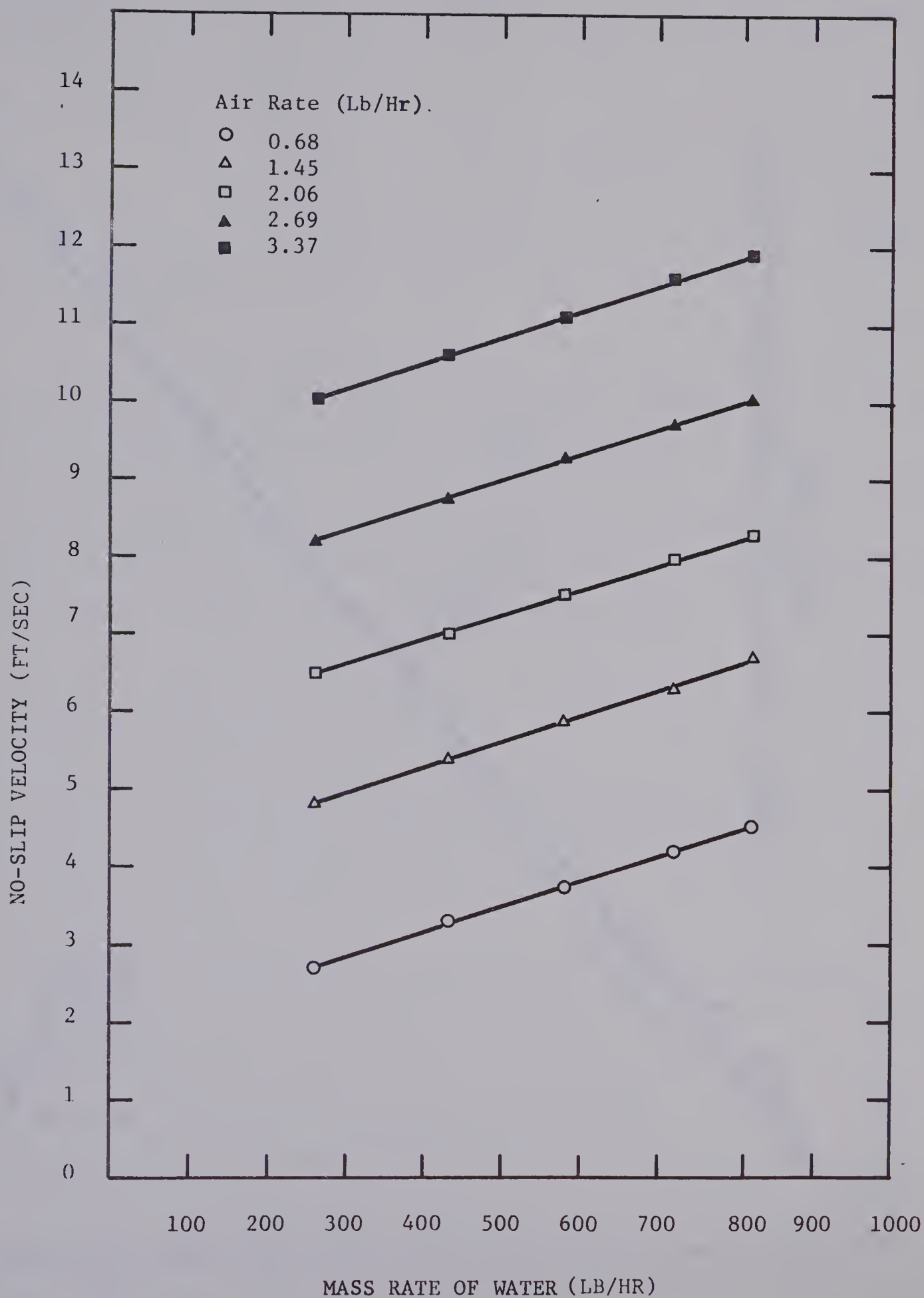
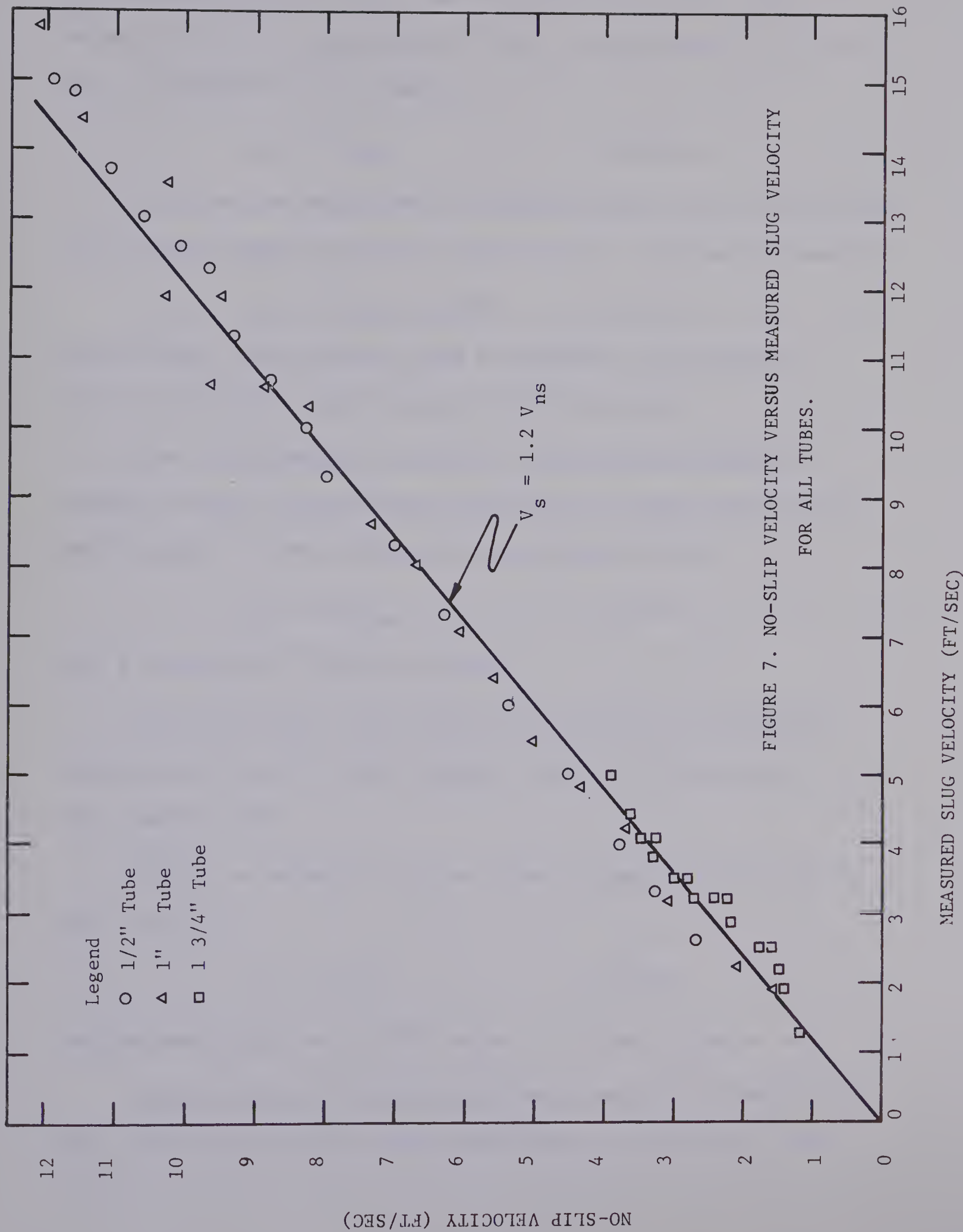


FIGURE 6. NO-SLIP VELOCITY VERSUS MASS RATE OF WATER FOR 1/2 INCH TUBE.



six determinations for a set of gas and liquid flow rates. The maximum deviation is approximately $\pm 10\%$. An approximate fit of the data is represented by the equation.

$$V_s = 1.2 V_{ns} \quad (5.2-1)$$

Nicolitsas and Murgatroyd (40) studied slug velocity distributions for air-water flows in vertical circular tubes. They used the equation

$$V_s = 1.2 V_{ns} + C_1 \sqrt{g D} \quad (5.2-2)$$

for air-slugs. This equation looks very similar to the equation (5.2-1) obtained for liquid slugs in horizontal tubes.

After the experimental portion of this work was completed, Gregory and Scott (25) published correlations of liquid slug velocity and frequency. A line of best fit through their data is

$$V_s = 1.35 V_{ns} \quad (5.2-3)$$

with a deviation of $\pm 18\%$ from the mean.

Gregory and Scott (25) carried out experiments on horizontal cocurrent slug flow for carbon dioxide - water in a three-quarter inch diameter tube.

Hubbard, as referred to in the paper by Gregory and Scott (25), gave a relation

$$V_s = 1.25 V_{ns} \quad (5.2-4)$$

for two-phase slug flow in 0.0351 meter (1 3/8 inch) diameter tube.

Comparison with the slug velocity measurements of Kordyban and Ranov (33) shows that the trend is very similar to the data. The

velocity of the slugs are a stronger function of the air flow rate than of the water flow rates.

Slug velocity measurements at different points in the test section by pressure measuring system did not show appreciable acceleration of slugs. Kordyban and Ranov also reported no noticeable acceleration of slugs over three 20-inch intervals.

5.2.2 Pressure Fluctuations

Pressure fluctuations represent at least to a first approximation the pressure drop across one slug. Other complicating factors such as slug acceleration, slug collapse, film holdup and the slug formation process have been observed. Sometimes subatmospheric pressures were produced by small fluctuations at low rates of flow of air and water. Several typical traces of pressure fluctuations are shown in Figure 8.

A log - log plot of pressure fluctuation data against no-slip velocity for all three tubes is shown in Figure 9. Data on pressure fluctuation measurements of Kordyban and Ranov (33) for the air-water system in one and one-quarter inch tube have also been plotted.

The curve shows that the magnitude of the pressure fluctuations is independent of the diameter of the tube at a large no-slip or superficial slug velocity. The data for the one and three-quarter inch tube does not fall on the average curve. However, most of the data for this tube falls in the plug flow region rather than in the slug flow region.

An approximate fit through the data gives a curve of slope 1.76. This shows that the pressure fluctuations vary directly with the slug velocity raised to a power 1.76 and are independent of pipe diameter.

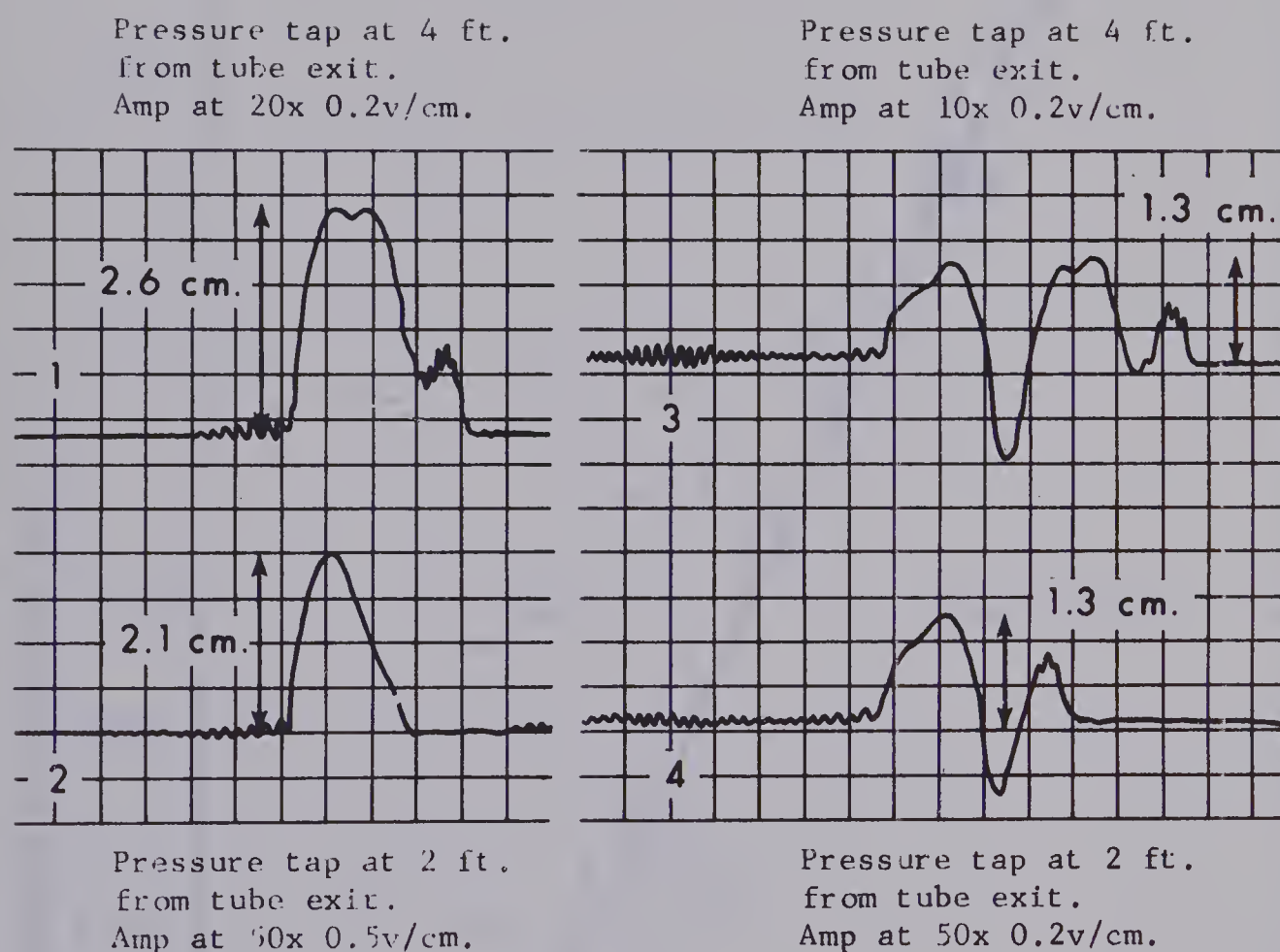
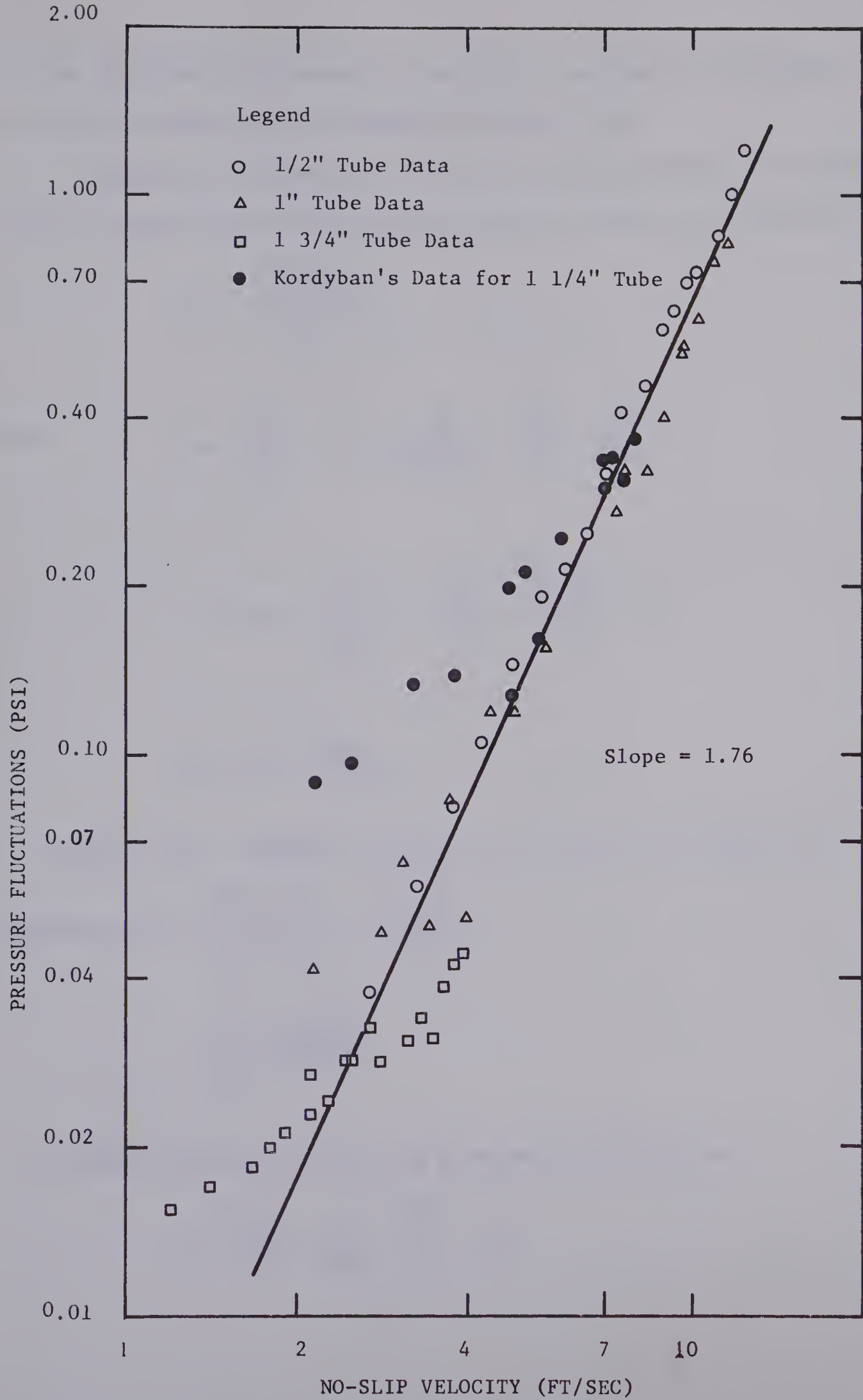


Chart Speed 25 mm/sec.

THE PRESSURE FLUCTUATIONS READ FROM THE CALIBRATION TABLES GIVEN IN APPENDIX D FOR THE ABOVE CHART ARE :

1. 18.0 CM WATER
2. 18.0 CM WATER
3. 6.0 CM WATER
4. 5.5 CM WATER

FIGURE 8. TYPICAL TRACES OF PRESSURE FLUCTUATIONS FOR THE ONE INCH TUBE.



This can be shown approximately by analysing the theory of Vermeulen (44) and the correlations of Gregory and Scott (25).

Neglecting the momentum term in the energy balance of Vermeulen (44) and assuming most of the pressure drop to occur due to viscous flow:

$$\Delta P_f \approx \frac{2f\rho V_s^2 L_{os}}{g_c D}$$

Also,

$$L_{os} = \frac{L_s}{N_s} = L_t \left(\frac{Q_L}{Q_L + Q_G} \right) \frac{\pi r^2}{\pi r^2} \frac{V_s}{N}$$

$$= L_t \left(\frac{Q_L}{Q_L + Q_G} \right) \frac{\pi r^2}{\pi r^2} \left(\frac{Q_L + Q_G}{\pi r^2} \right) \frac{1}{N}$$

$\therefore$

$$L_{os} = L_t \frac{Q_L}{\pi r^2 N}$$

Assuming, $\rho V_s \approx \text{constant} = C_1$ for $V_{ns} > 3\text{ft/sec}$, (see Figure 25)

Therefore, $\Delta P_f = \frac{2f V_s^2 C_1}{g_c D V_s} \frac{L_t Q_L}{\pi r^2 N}$

$$= \frac{2C_1 f L_t Q_L V_s}{g_c D \pi r^2 N}$$

By Gregory and Scott (25), slug frequency is defined as:

$$N = \frac{Q_L}{\pi r^2} \frac{1}{Dg} \left(\frac{36}{V_s} + V_s \right),$$

where V_s is in metre/sec.

If values of V_s are very small, $\frac{36}{V_s} + V_s \approx \frac{36}{V_s}$

Thus,
$$N = \frac{36 Q_L}{\pi r^2 D g V_s}$$

Therefore,

$$\Delta P_f = \frac{2 C_1 f L_t Q_L V_s}{g_c D \pi r^2 N} \frac{\pi r^2 D g V_s}{36 Q_L}$$

$$= C_2 f V_s^2, \text{ where } C_2 = \text{a constant.}$$

Also $f = 0.0791/R_e^{0.25}$

$= C_3/V_s^{0.25}$, C_3 = a constant depending only on the tube size and physical properties of the fluids used.

Now,

$$\Delta P_f \approx (C_2 V_s^2) (C_3/V_s^{0.25})$$

i.e. $\Delta P_f = C V_s^{1.75}$, where C = a constant.

Thus from the slug theory of Vermeulen (44) and the correlation of Gregory and Scott (25), the pressure fluctuations are approximately independent of tube diameter at low velocity and are directly proportional to the slug velocity raised to a power 1.75. From the data an exponent of 1.76 is obtained, showing good agreement with the above.

5.2.3 Pressure Drop

Representative pressure drop data have been presented for one-half, one, and one and three-quarter inch tubes in Figures 10 to 18. Complete data are included in Appendix A. The figures also show a comparison of data with the pressure drop correlations of Lockhart-

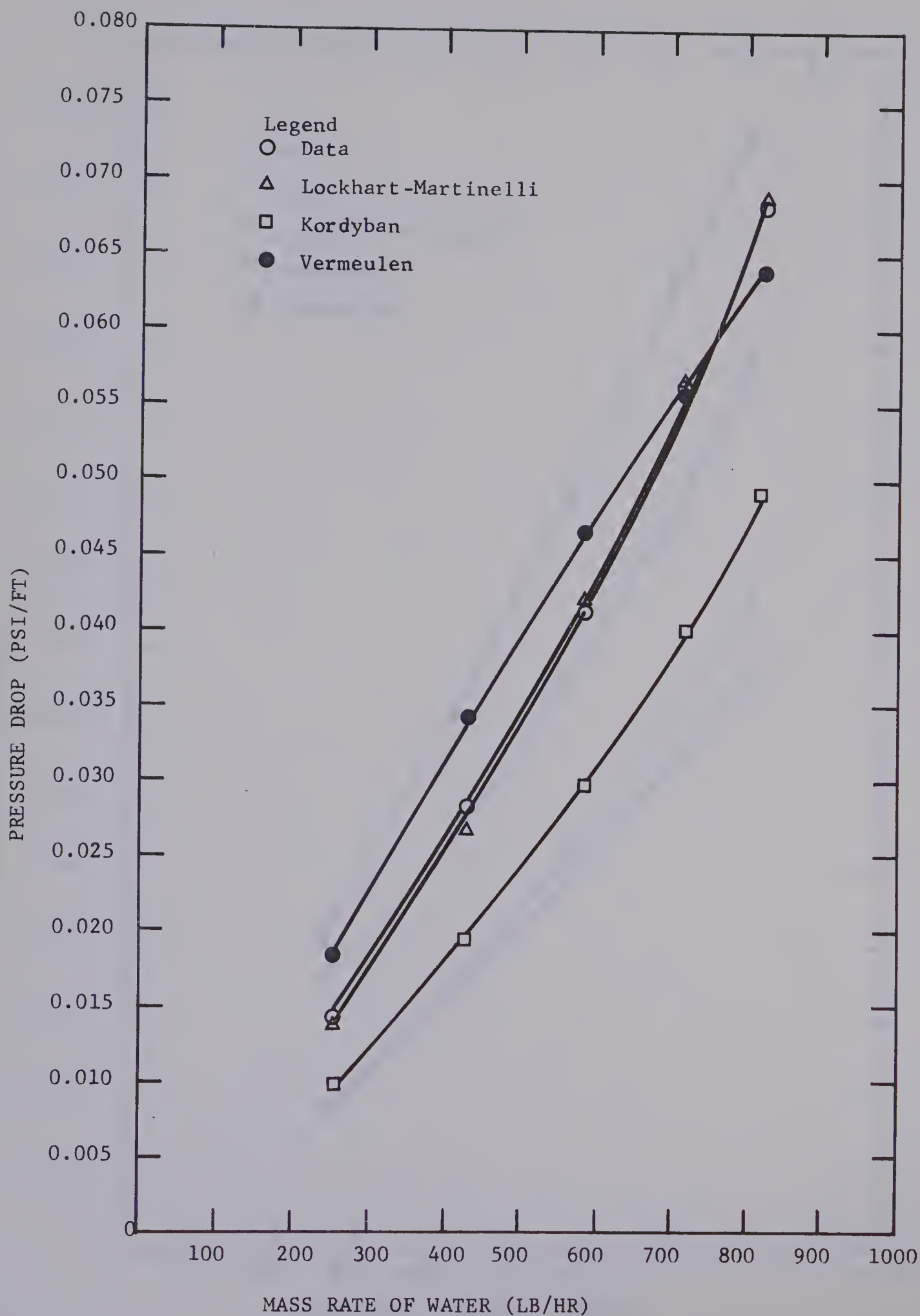


FIGURE 10. COMPARISON OF THE PRESSURE DROP CORRELATIONS WITH THE DATA AT 0.68 LB/HR AIR RATE FOR 1/2 INCH TUBE.

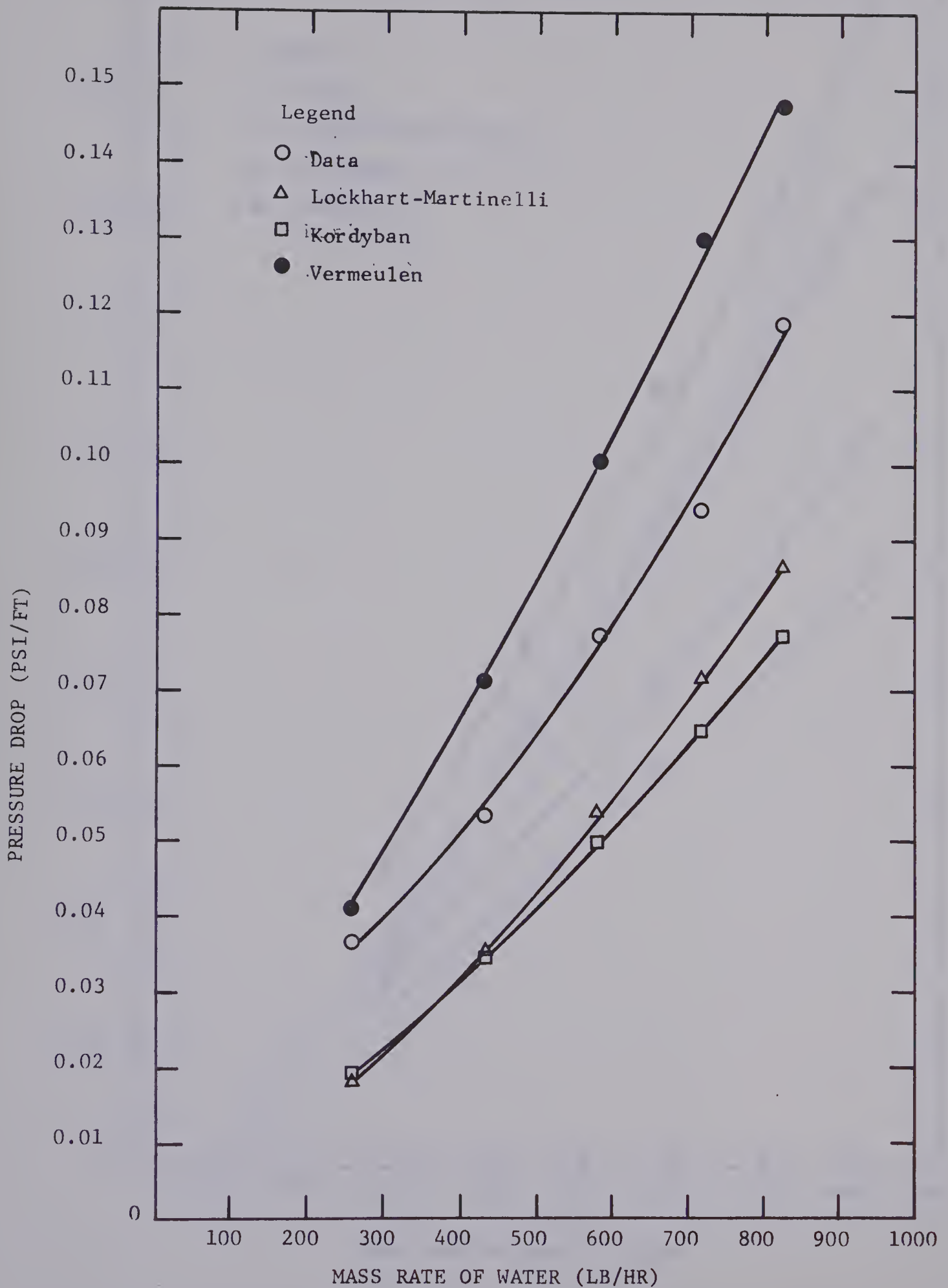


FIGURE 11 COMPARISON OF THE PRESSURE DROP CORRELATIONS WITH THE DATA AT 2.06 LB/HR AIR RATE FOR 1/2 INCH TUBE.

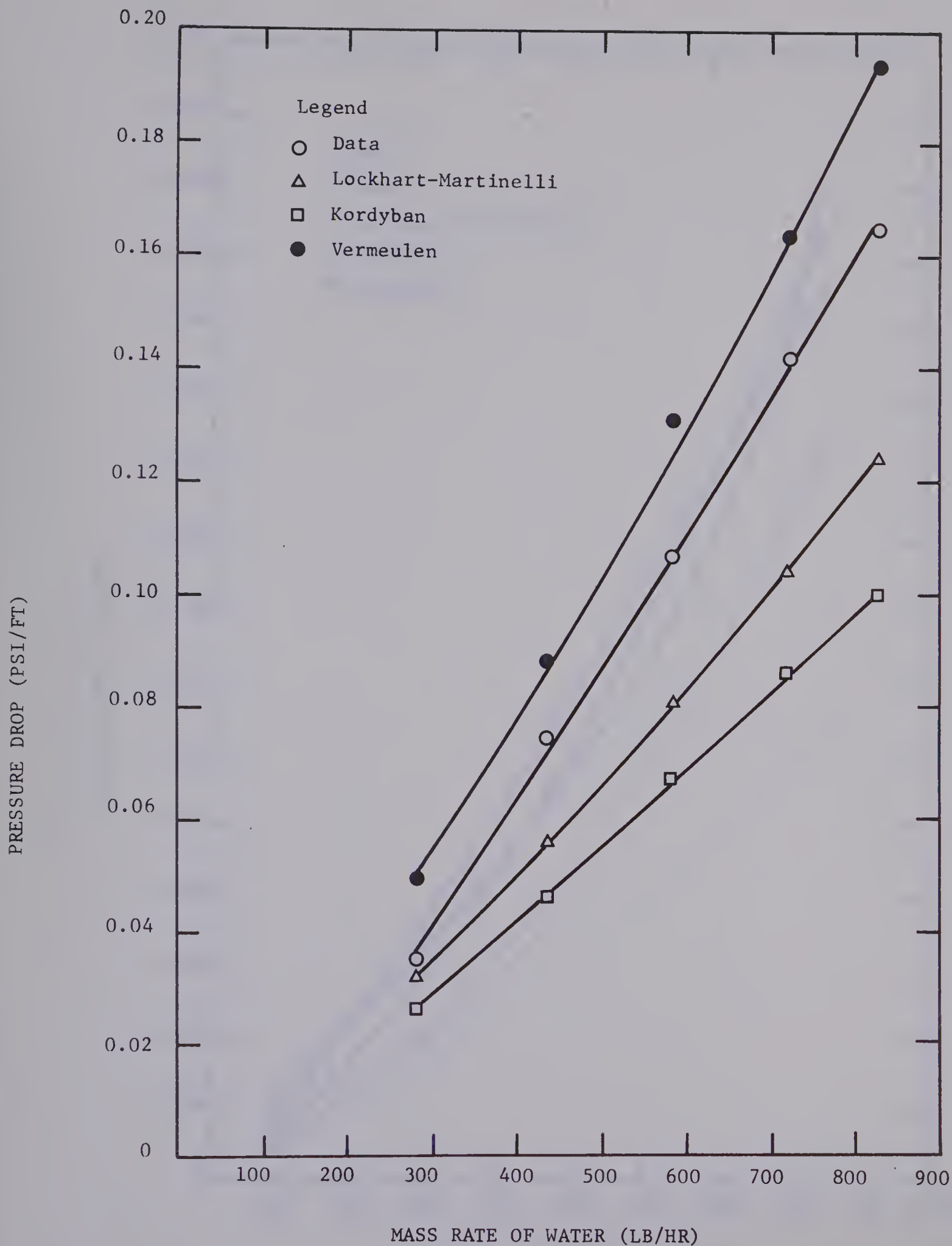


FIGURE 12. COMPARISON OF THE PRESSURE DROP CORRELATIONS WITH THE DATA AT 3.37 LB/HR AIR RATE FOR 1/2 INCH TUBE.

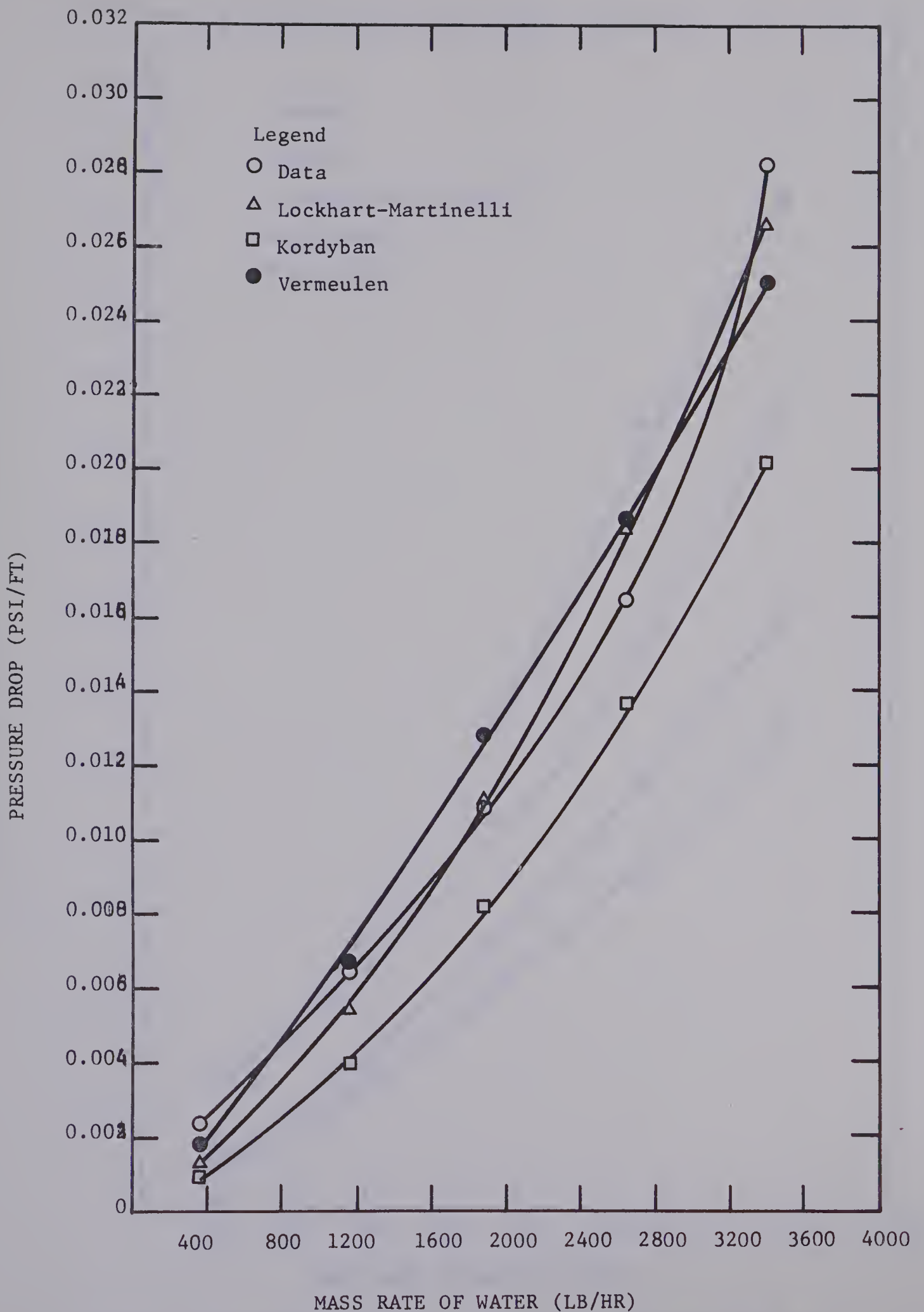


FIGURE 13. COMPARISON OF THE PRESSURE DROP CORRELATIONS WITH THE DATA AT 1.79 LB/HR AIR RATE FOR 1 INCH TUBE.

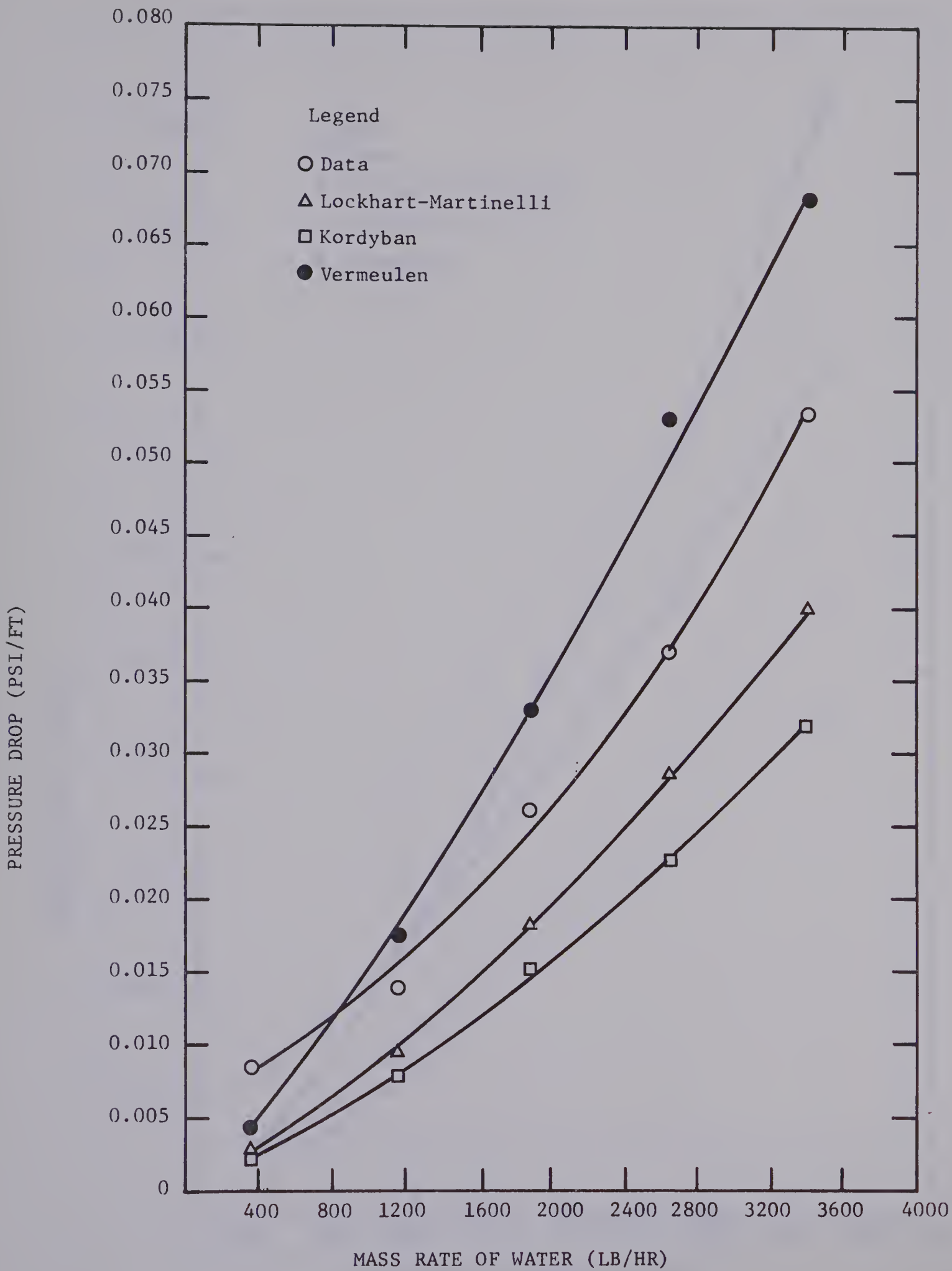


FIGURE 14. COMPARISON OF PRESSURE DROP CORRELATIONS WITH THE DATA AT 6.66 LB/HR AIR RATE FOR 1-INCH TUBE.

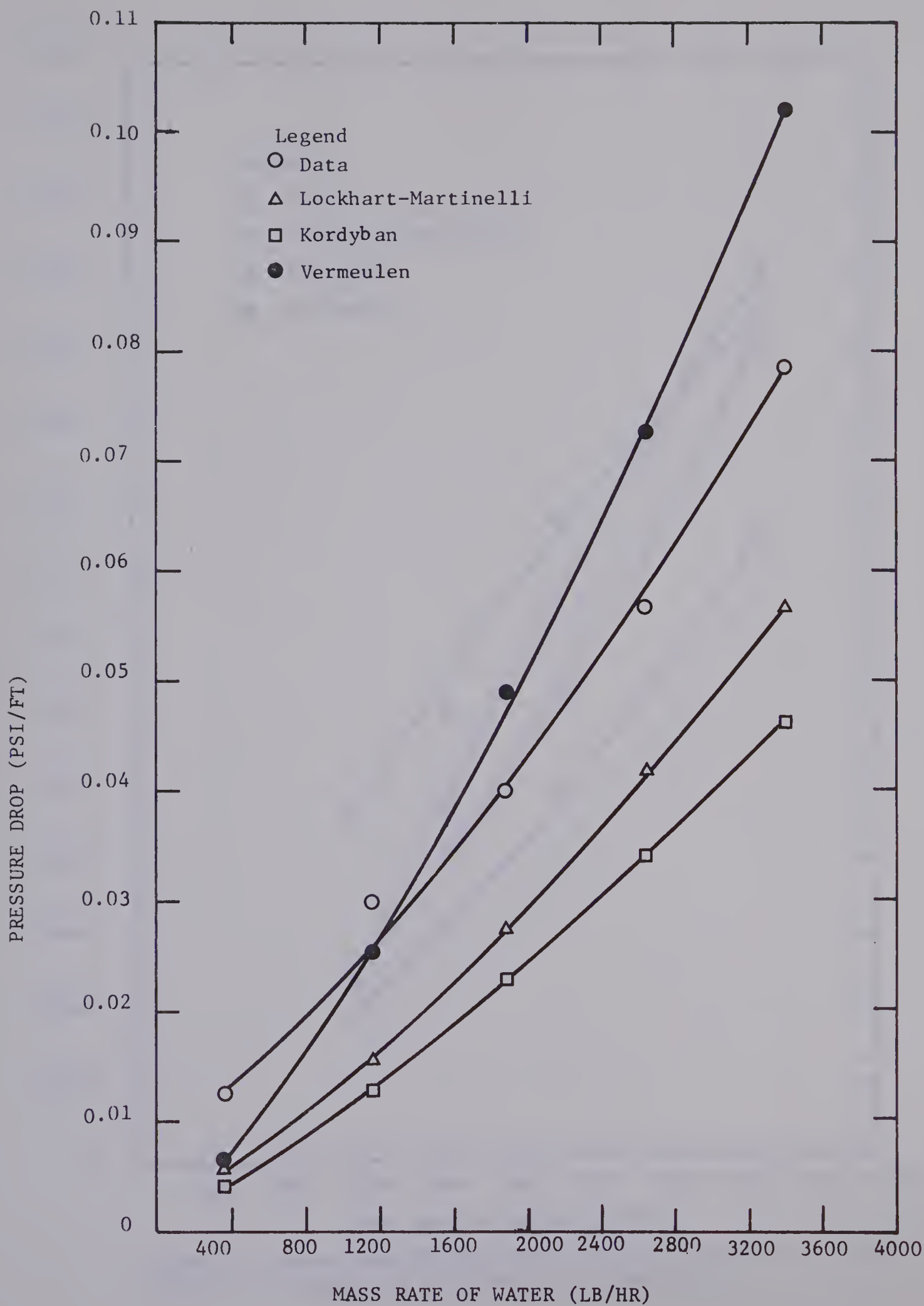


FIGURE 15. COMPARISON OF PRESSURE DROP CORRELATIONS WITH THE DATA AT 13.64 LB/HR AIR RATE FOR 1-INCH TUBE.

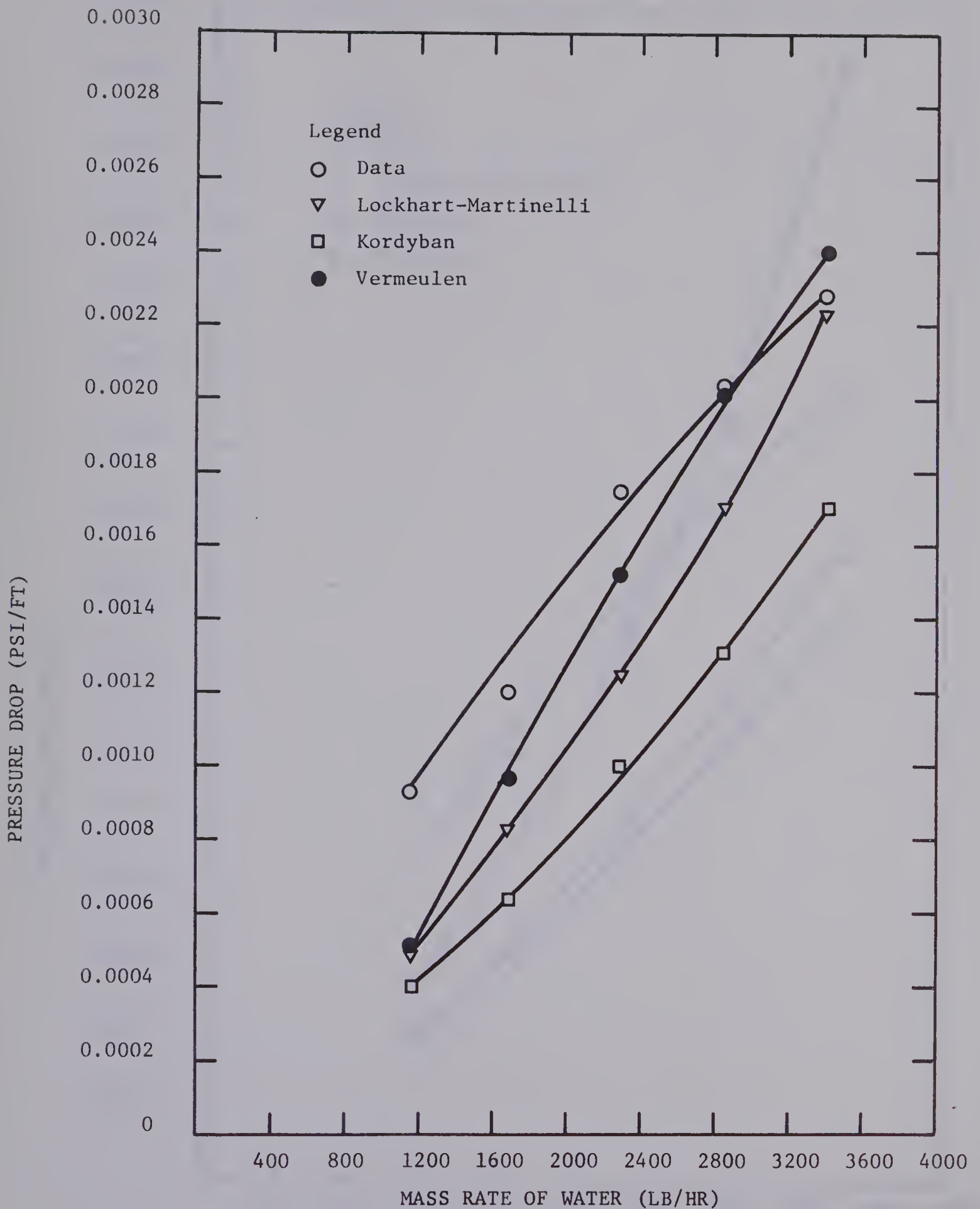


FIGURE 16. COMPARISON OF PRESSURE DROP CORRELATIONS WITH THE DATA AT 4.11 LB/HR AIR RATE FOR 1 3/4 INCH TUBE.

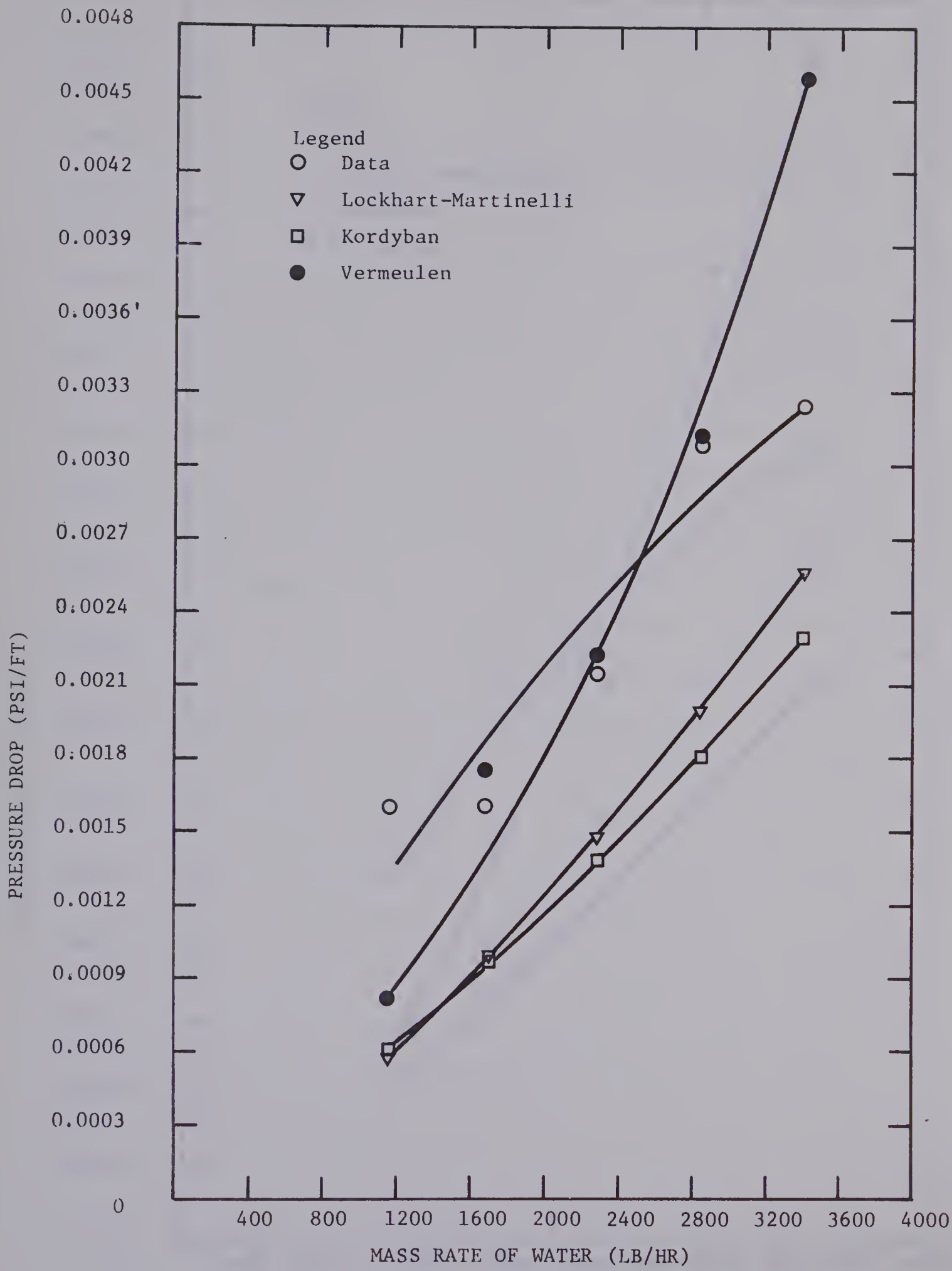


FIGURE 17. COMPARISON OF PRESSURE DROP CORRELATIONS WITH THE DATA AT 8.12 LB/HR AIR RATE FOR 1 3/4 INCH TUBE.

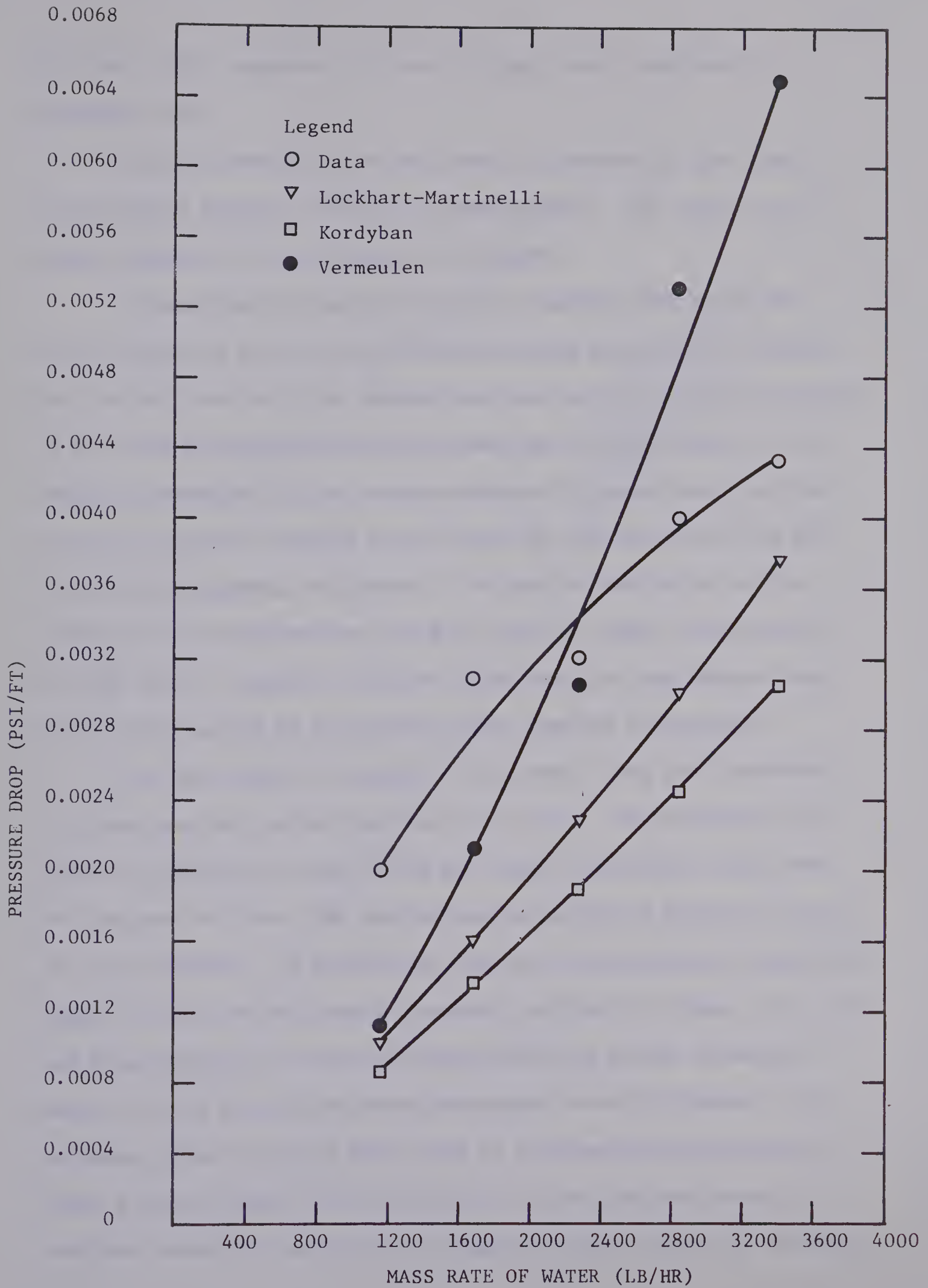


FIGURE 18. COMPARISON OF PRESSURE DROP CORRELATIONS WITH THE DATA AT 13.64 LB/HR AIR RATE FOR 1 3/4 INCH TUBE.

Martinelli (36), Kordyban (32) and the slug theory developed by Vermeulen (44).

For all three tubes, at any rate, an increase in the liquid rate causes a gradual increase in pressure drop. The slope of the curves increase as the air rate is increased.

A comparison of the predictions of Lockhart-Martinelli and that of Kordyban with the data shows that good agreement is obtained for low air rates only for one-half and one inch tubes. With increase of air rate the deviations become marked for all three tubes. The range of deviations of the Lockhart-Martinelli predictions from the data is 15-40% for one-half inch, 20-70% for one inch and 5-50% for one and three-quarter inch tubes. The range of deviations of the predictions of Kordyban from the data is still higher than these for all the tubes. Kordyban (32) also found that his experimental data points fell some 20 to 30 percent higher than his correlation.

The slug model of Vermeulen (44) shows a very good agreement with the data for one-half and one inch tubes. The predictions lie within 25 percent for most of the air rates for one-half inch tube. For the one inch tube, the predictions are within 40 percent for most of the air rates. The predictions from the slug model are consistently higher for most of the data for one-half and one inch tubes. For one and three-quarter inch tube the predictions are within 45 percent except for low water rates where deviations are still higher. For low water rates the slugs were found to collapse and when plotted on Baker's chart (Figure 19), the data lie in the plug region and in the lower range of "intermittent slugging" region defined by Vermeulen

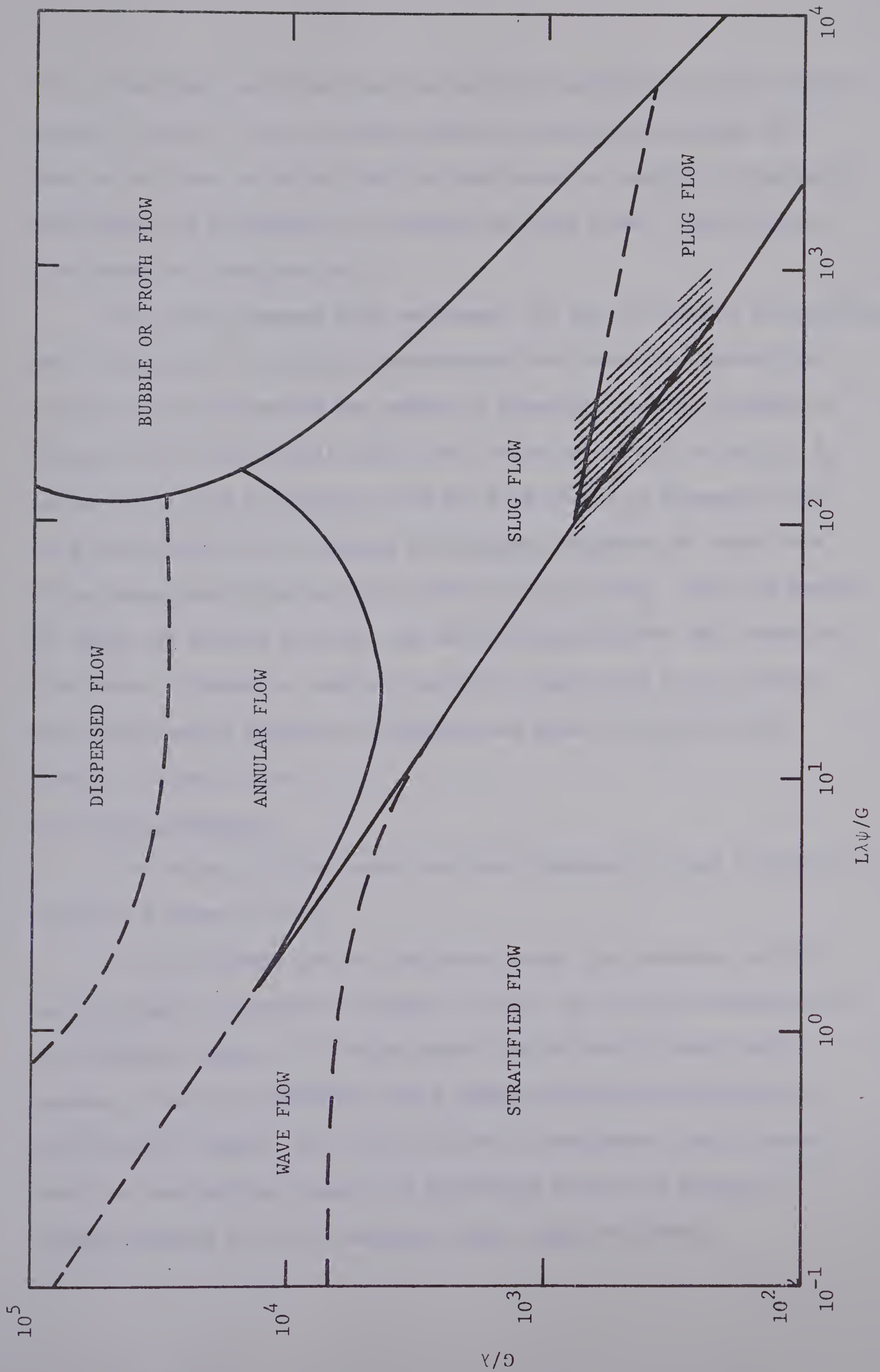


FIGURE 19. COMPARISON OF BAKER'S FLOW PATTERN WITH THE DATA IN 1 3/4 INCH TUBE

(44). The slug theory would not be strictly applicable in that region. However, the fact that the predictions are consistently higher for most of the data indicates that the slug model is superior to Lockhart-Martinelli and Kordyban as it provides an upper bound. This fact was also noted by Vermeulen (44).

The total pressure drop represents the sum of pressure fluctuations over the section. A plot of pressure gradient versus the product of pressure fluctuations and the number of slugs per section is shown in Figure 20 for the one-half inch tube. The relation $\Delta P/L = (\Delta P_f/L) N_s$ holds fairly well as expected from the slug theory of Vermeulen (44). As stated earlier, the pressure fluctuations represent at least to a first approximation the pressure drop across one slug. Also the number of slugs per section for one, and one and three-quarter inch tubes are fractional. Therefore, similar plots for these tubes are not shown, since the overall pressure is discrete and equal to ΔP_f when the number of slugs is one.

5.2.4 Slug Frequency

The effect of fluid rates and tube diameters on slug frequency is shown in Figures 21 to 23.

It is observed that at low water rates, the frequency is low and it remains practically constant for one, and one and three-quarter inch diameter tubes. At low air rates the increasing water rates produce a sharp increase and then a steady decrease in frequency for all the three tubes. But in the one and three-quarter inch diameter tube, at the high air rates, the increasing water rate produces a steady increase after the decrease unlike other two tubes.

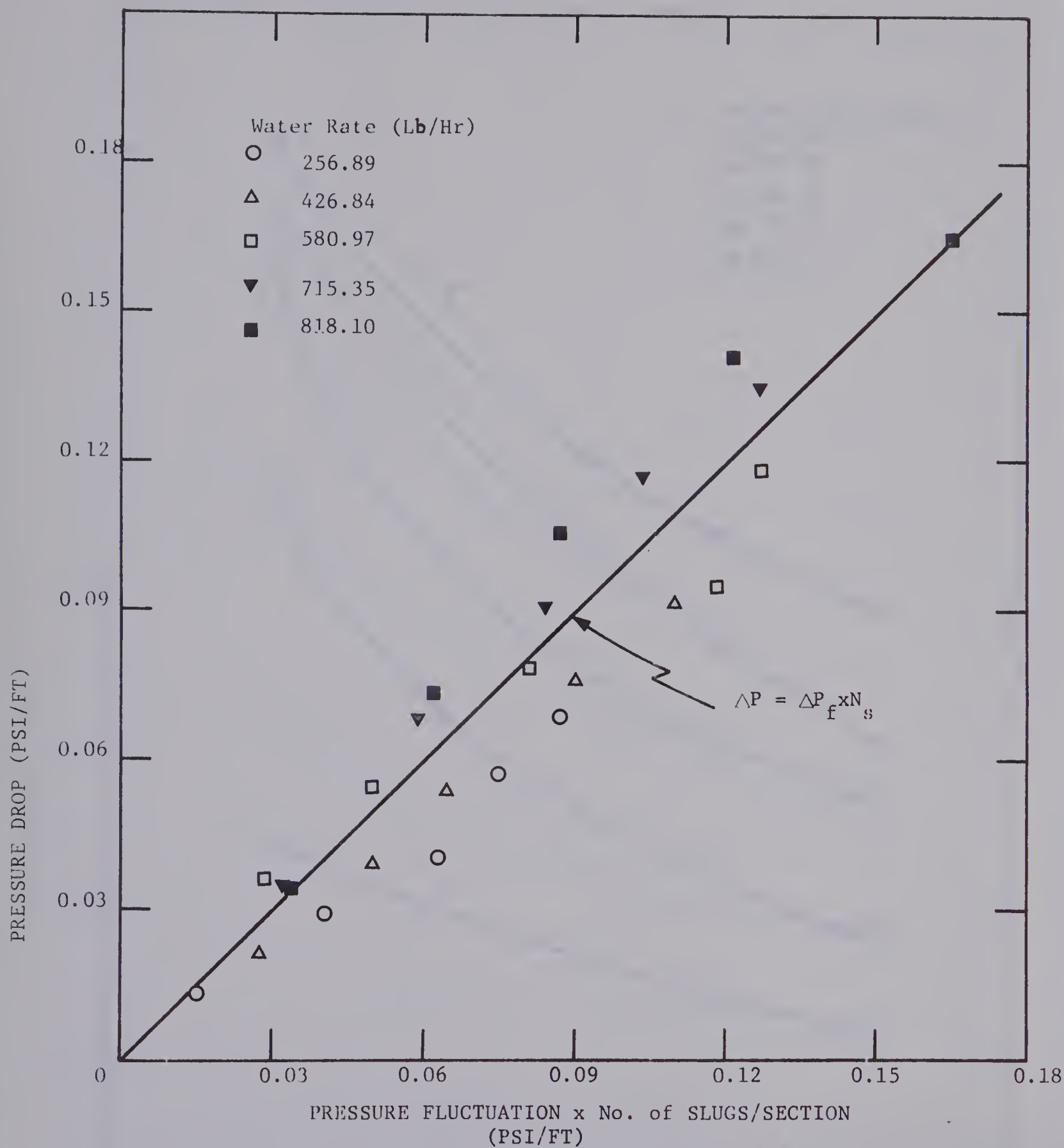


FIGURE 20. PRESSURE DROP VERSUS (PRESSURE FLUCTUATION) x (No. OF SLUGS/SECTION) FOR 1/2 INCH TUBE.

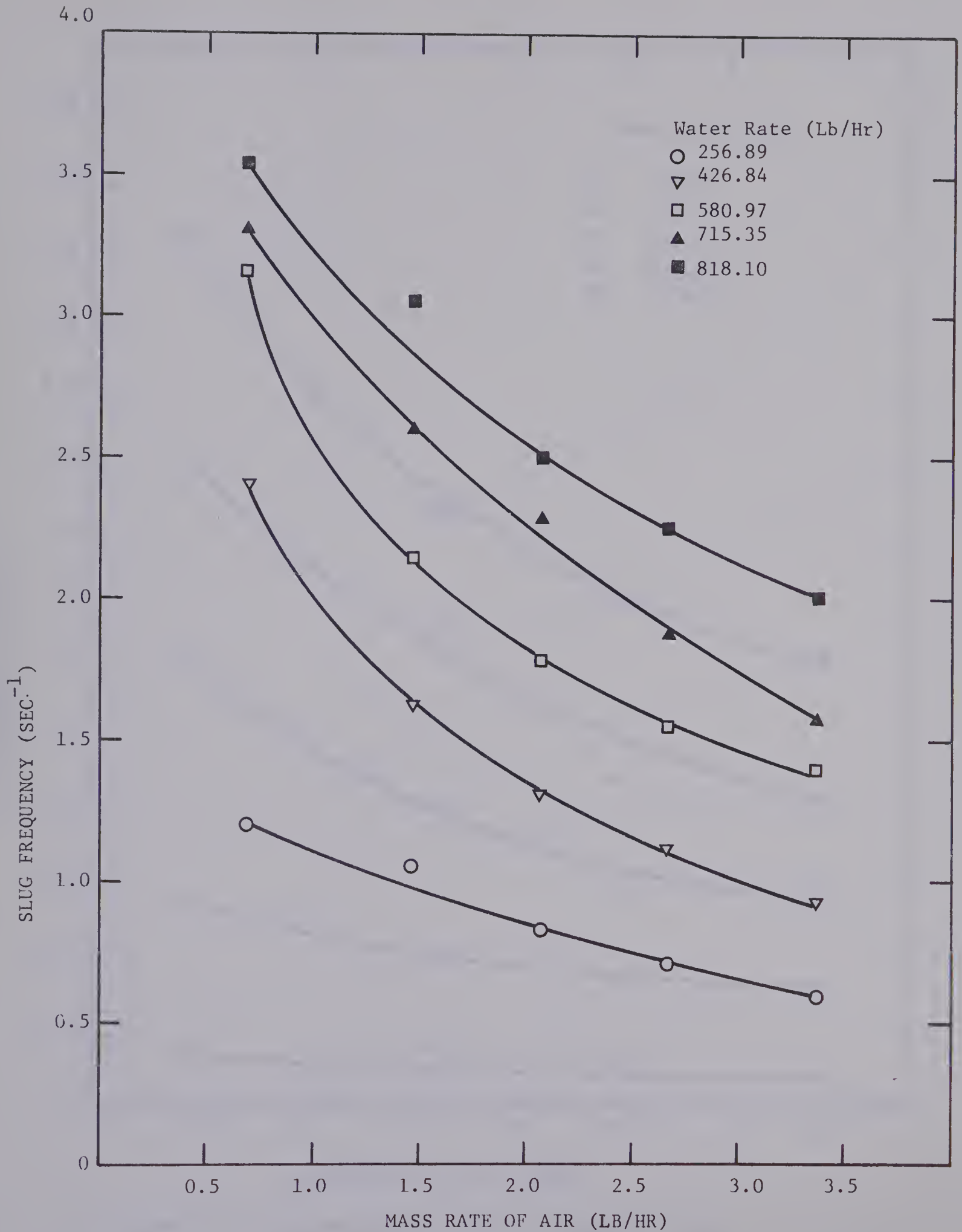


FIGURE 21. SLUG FREQUENCY VERSUS MASS RATE OF AIR FOR 1/2 INCH TUBE.

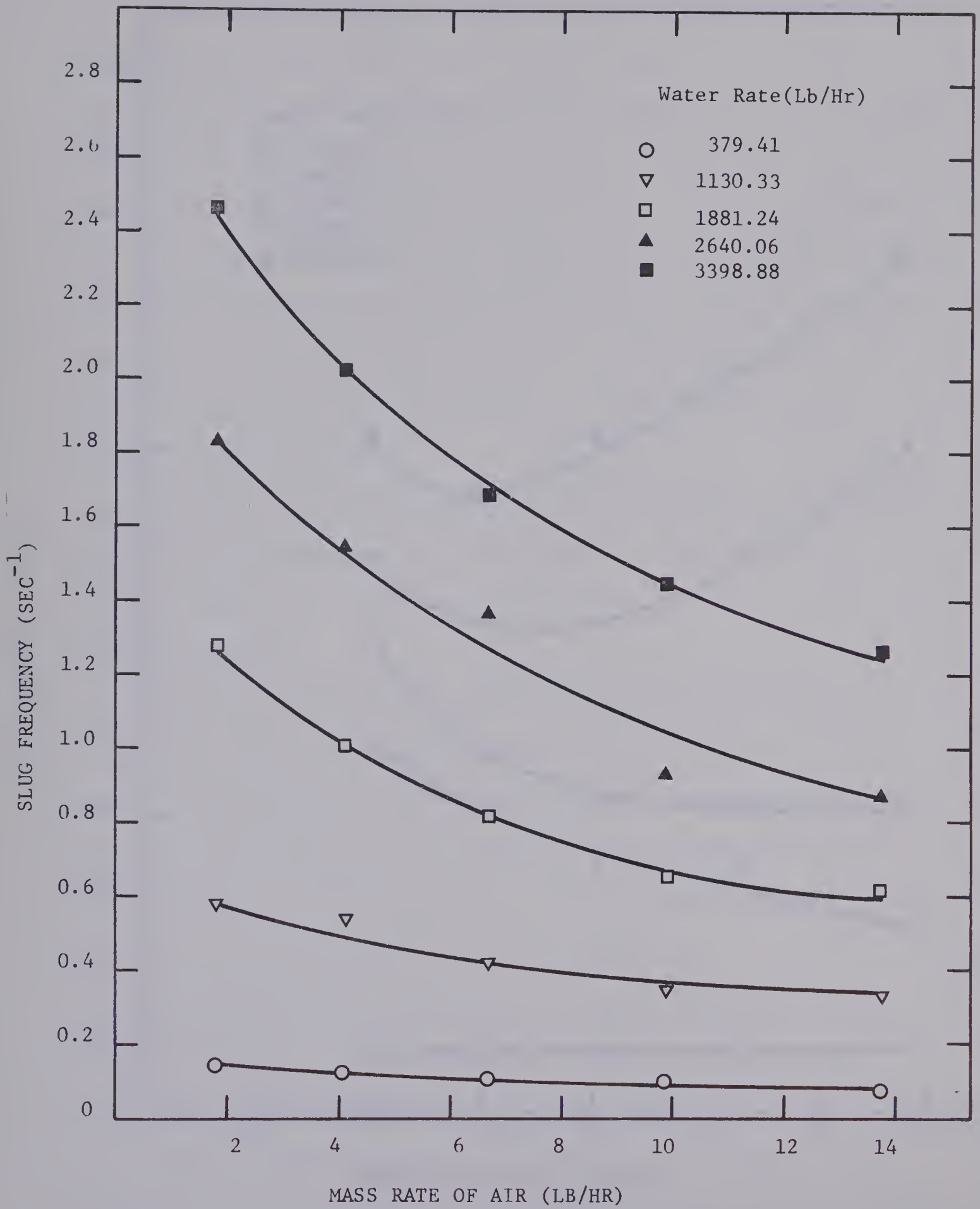


FIGURE 22. SLUG FREQUENCY VERSUS MASS RATE OF AIR
FOR 1 - INCH TUBE.

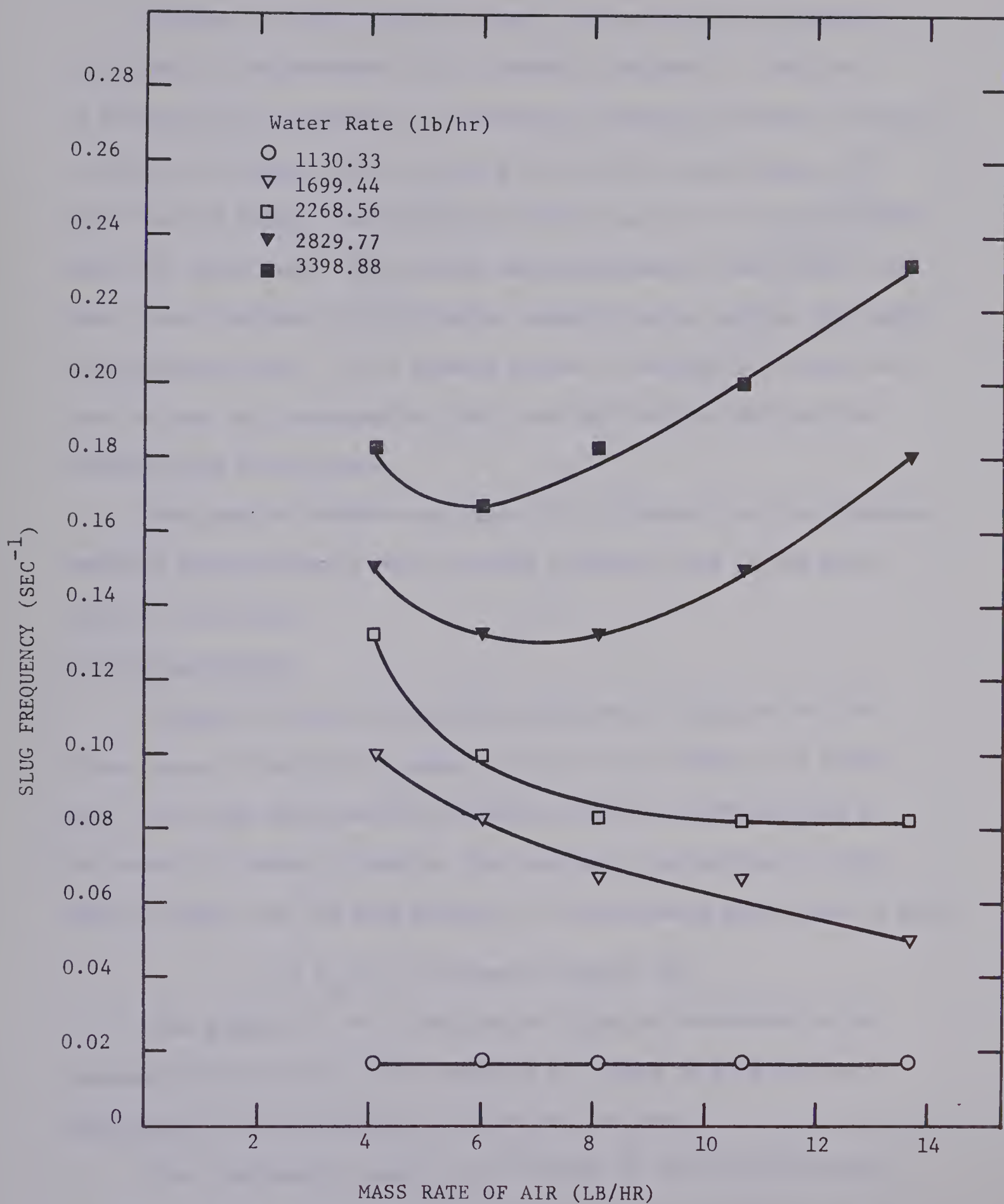


FIGURE 23. SLUG FREQUENCY VERSUS MASS RATE OF AIR FOR 1 3/4 INCH TUBE.

Further, a plot of slug frequency versus slug Froude number as defined by Gregory-Scott (25) is shown in Figure 24. The data of Vermeulen (44), Hubbard (as presented by Gregory and Scott) and that of Gregory and Scott (25) are also plotted on the same figure. As seen from the figure, the data for one-half and one inch tubes compare well with their data. For the one and three-quarter inch tube at low water rates the data do not agree, possibly being outside the region of steady slug flow. It is already stated in section 5.2.3 that the data for one and three-quarter inch tube fall in the plug and the unsteady slug flow region.

The work of Kordyban and Ranov (33), although far from complete, compares reasonably well with the slug frequency data in the same region of slug flow.

5.2.5 Slug Density

Figure 25 shows the measured densities of slugs for all the three tubes. This gives a measure of air in the slugs or of frothy flow. The slug densities are evaluated from the determinations of the amount of liquid in slug at the tube exit, the percent of total time on slugs, and the slug velocity. The following expression is used :

$$\rho_s = Q_s / (\pi r^2) (\% \text{ time on slugs}) (V_s)$$

The percent of the total time on slugs is determined by the conductivity probe data. The fraction of liquid in slug and the slug velocity are determined as described earlier.

The slug density seems to be related to the relative motion between the slug and the film. The figure shows that the slug density

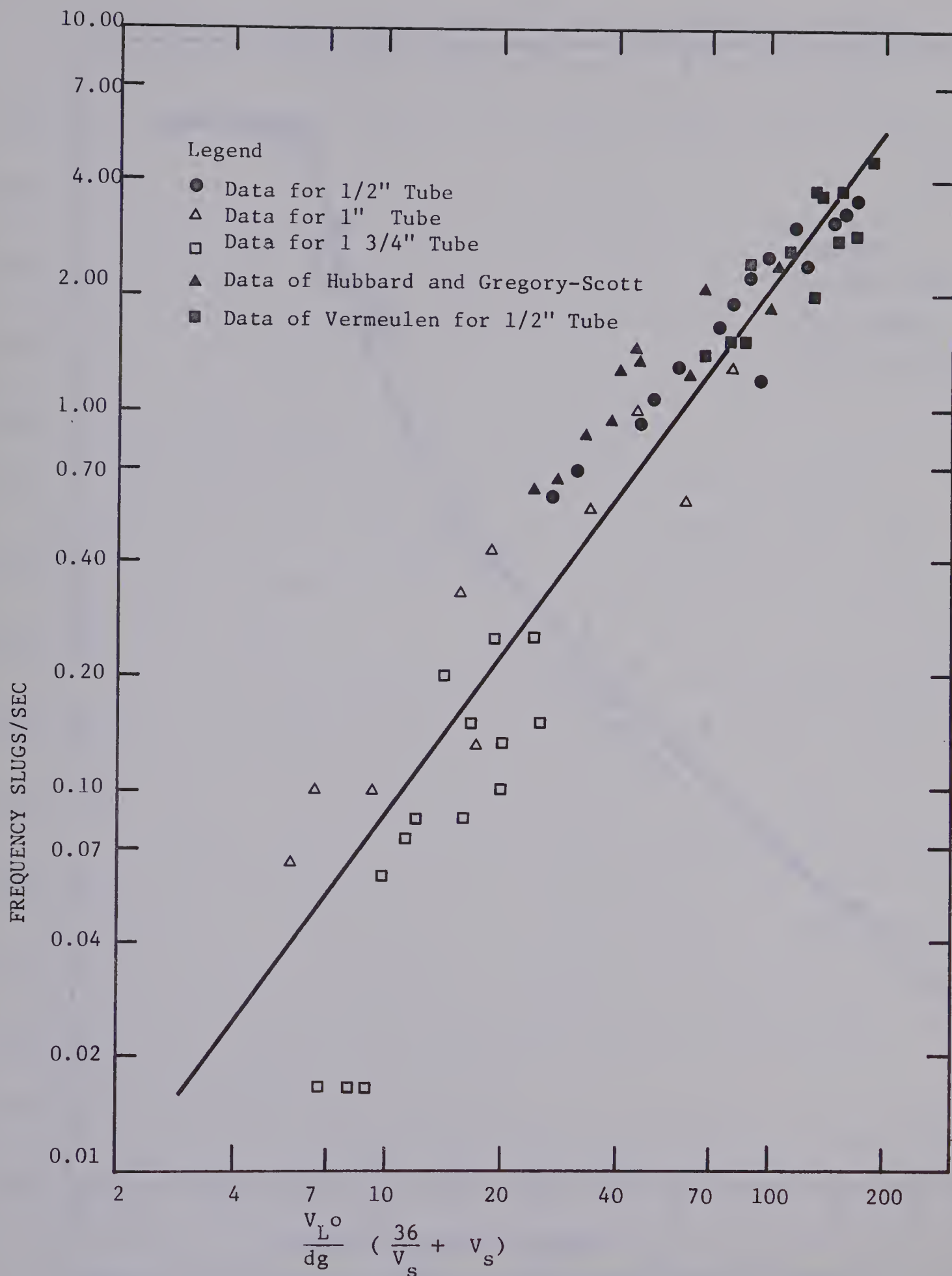


FIGURE 24. SLUG FREQUENCY VERSUS SLUG FROUDE NUMBER AS DEFINED BY GREGORY-SCOTT FOR ALL TUBES.

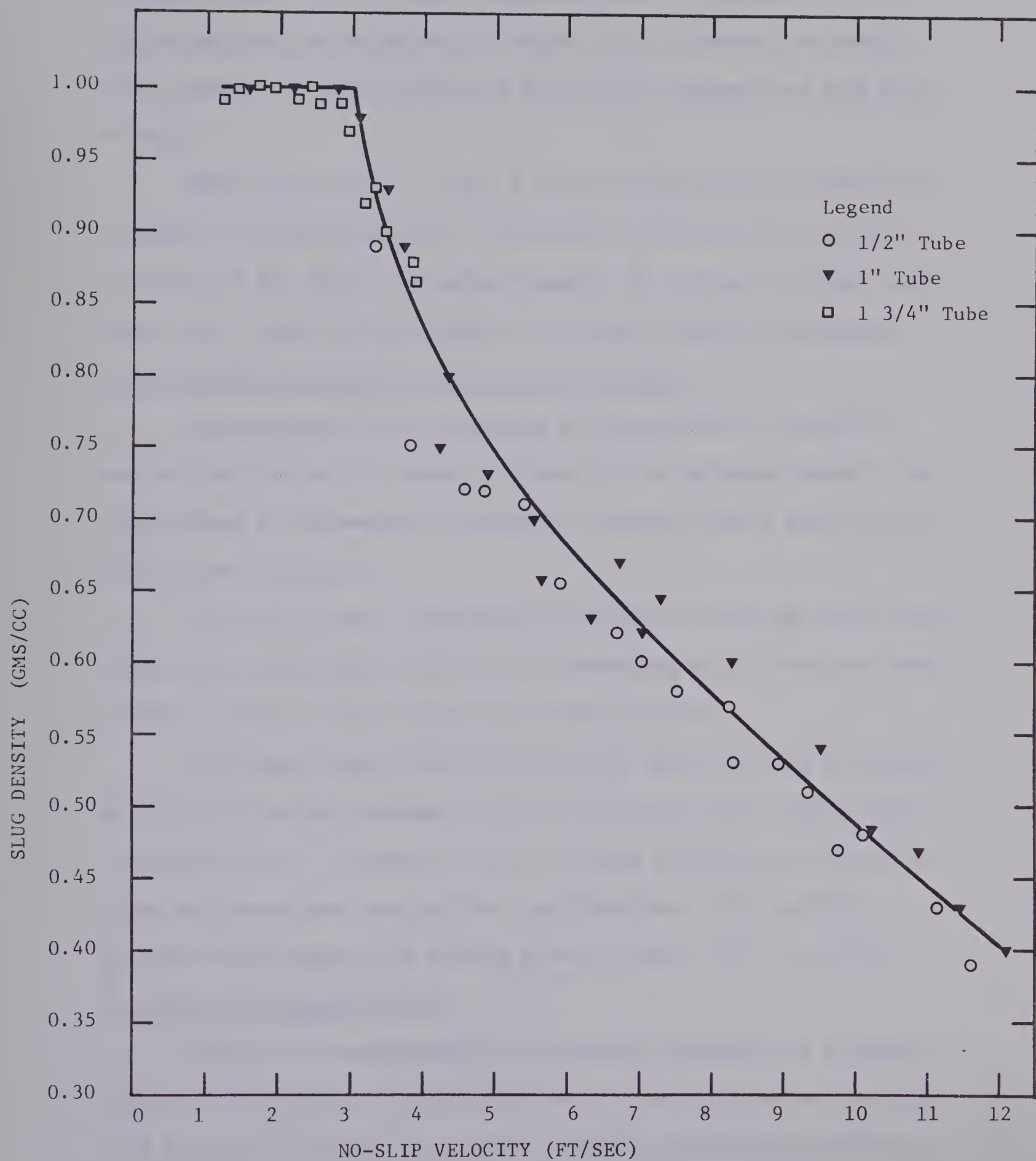


FIGURE 25. SLUG DENSITY VERSUS NO-SLIP VELOCITY FOR ALL TUBES.

is independent of tube diameter depending only on velocity. At low slug velocities and hence low air rates, the slug density is nearly that of water. At high velocities the density becomes less than that of water.

When extrapolated to $\rho_s = 0$, a value of approximately 20ft/sec is obtained for no-slip velocity. This may be fortuitous but it gives the velocity for minimum slugging frequency as defined by Gregory and Scott (25). Data of slug density has not been reported previously.

5.2.6 Fraction of liquid in Slug at the tube exit

The magnitude of the variation in the fraction of liquid in slug at the tube exit is shown in Figure 26 for different tubes. The figure shows a plot against the ratio of the total fluid flow to the liquid flow, $(Q_L + Q_G)/Q_L$.

At low air rates, the fraction of liquid in slug is high, between 0.80-0.95, it decreases steadily with increasing water rates and levels off to a value of 0.65 at high air and water rates.

The figure shows that the fraction of liquid in slug is independent of tube size and dependent only on the ratio of the total fluid to the liquid flow. The value of 0.65 of liquid fraction in slug may be taken as the minimum required for slug formation. The remaining fraction being liquid film flowing from the tube is not the holdup.

5.3 Film Flow Characteristics

Fulford (21) summarised the information available on a number of aspects of film flow. A significant part of work carried out on film flow has been concerned in one way or another with the wavy nature of flowing film and with the interaction between film and adjacent gas

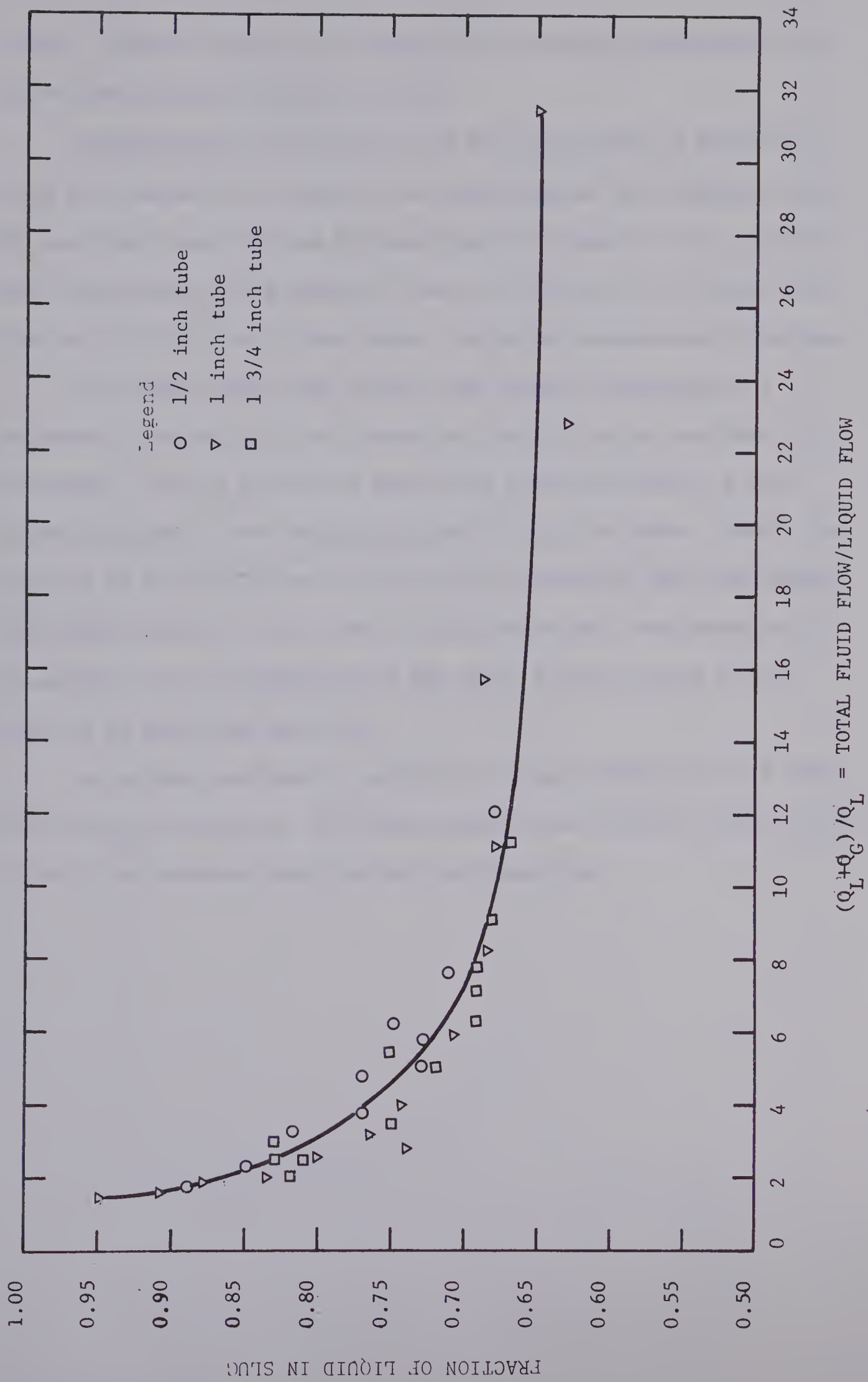


FIGURE 26. FRACTION OF LIQUID IN SLUG VERSUS TOTAL FLUID FLOW/LIQUID FLOW.

stream. Fulford referred to a method for obtaining instantaneous and undisturbed velocity profile in film.

Representative film flow curves have been shown in Figures 27 to 29 for one-half, one, and one and three-quarter inch diameter tubes. The data have been obtained by measuring the volumetric flow rates in each compartment of the separator box at different air and water rates. This is the only type of data taken. No holdup measurements were made.

The figure shows that as the film velocity increases, the increase in the ratio of the cumulative liquid flow to the total flow is gradual. But at a velocity where slug formation begins, a very sudden increase in the cumulative flow of liquid is noted. Again, the fraction of the total flow at various film velocities does not change appreciably except at the onset of slug formation. This behaviour is consistent for all diameter tubes and gives a good picture of the build up of slug from the film.

An attempt was made to calculate average pressure drop of films from the film flow data. The film pressure loss totalled about 10-20% of the total pressure drop for the two-phase flow.

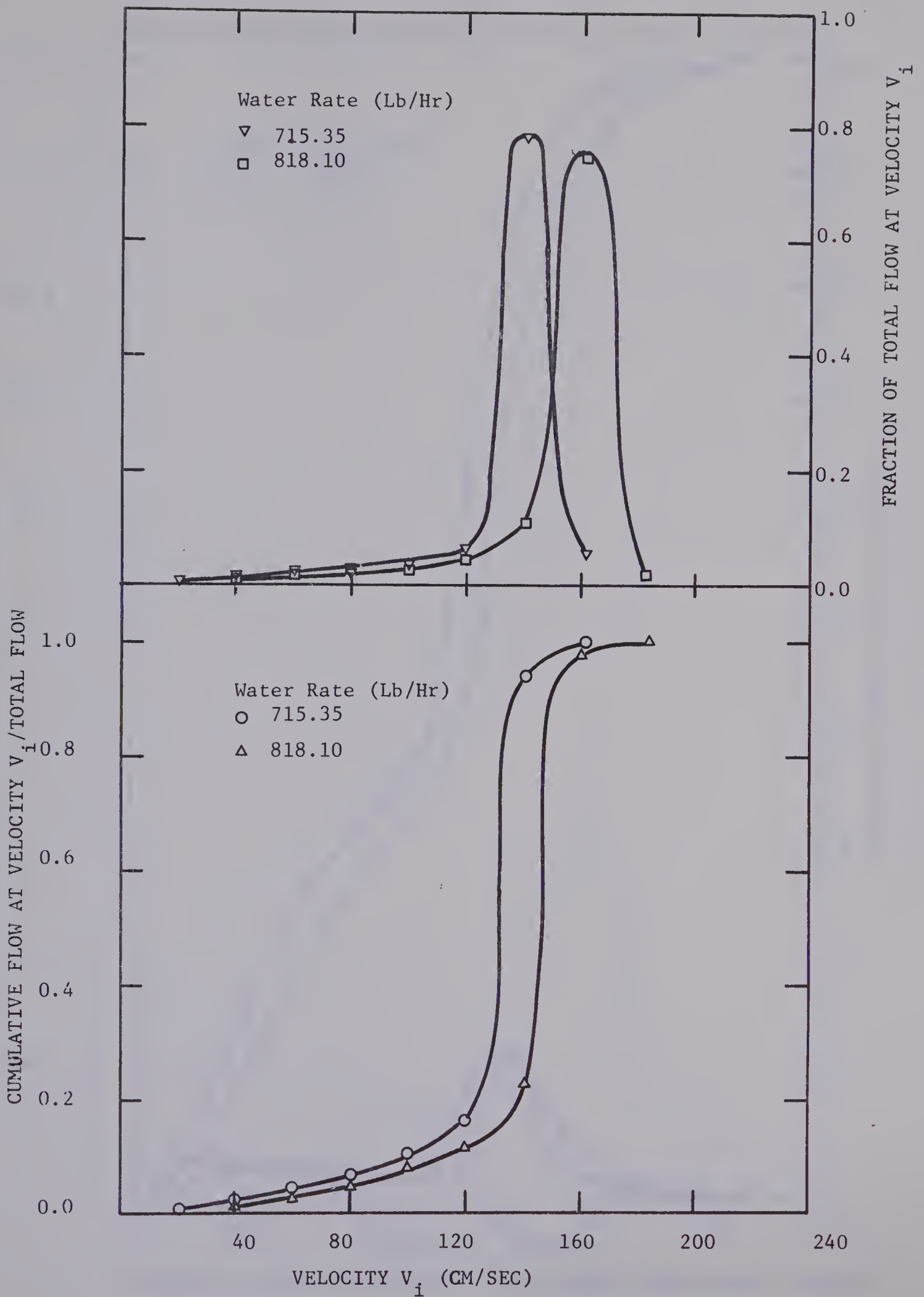


FIGURE 27. CUMULATIVE FLOW AND FRACTIONAL FLOW VERSUS VELOCITY AT AIR RATE OF 0.68 LB/HR FOR 1/2" TUBE.

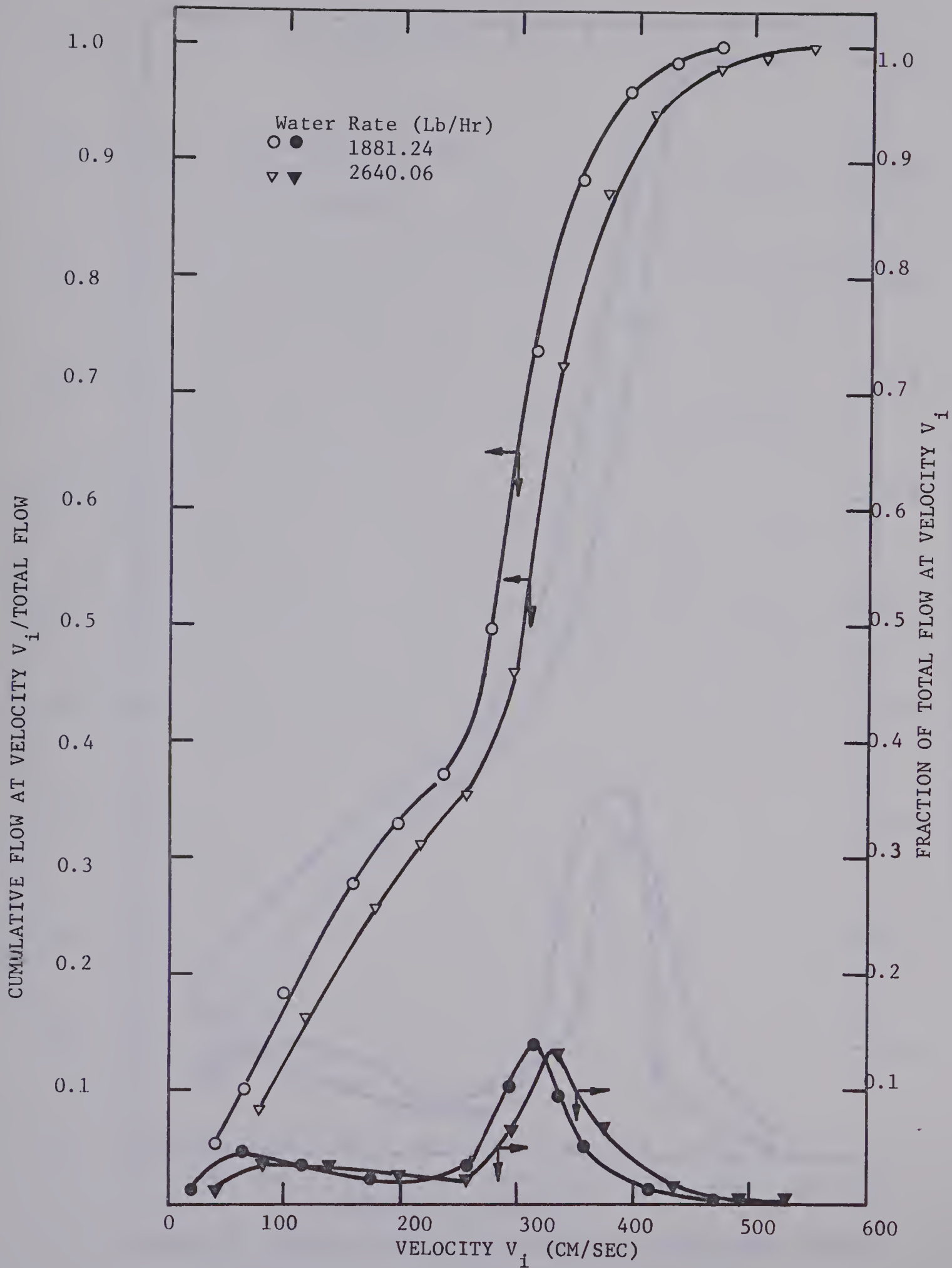


FIGURE 28. CUMULATIVE FLOW AND FRACTIONAL FLOW VERSUS VELOCITY AT AIR RATE OF 6.66 LB/HR FOR 1" TUBE.

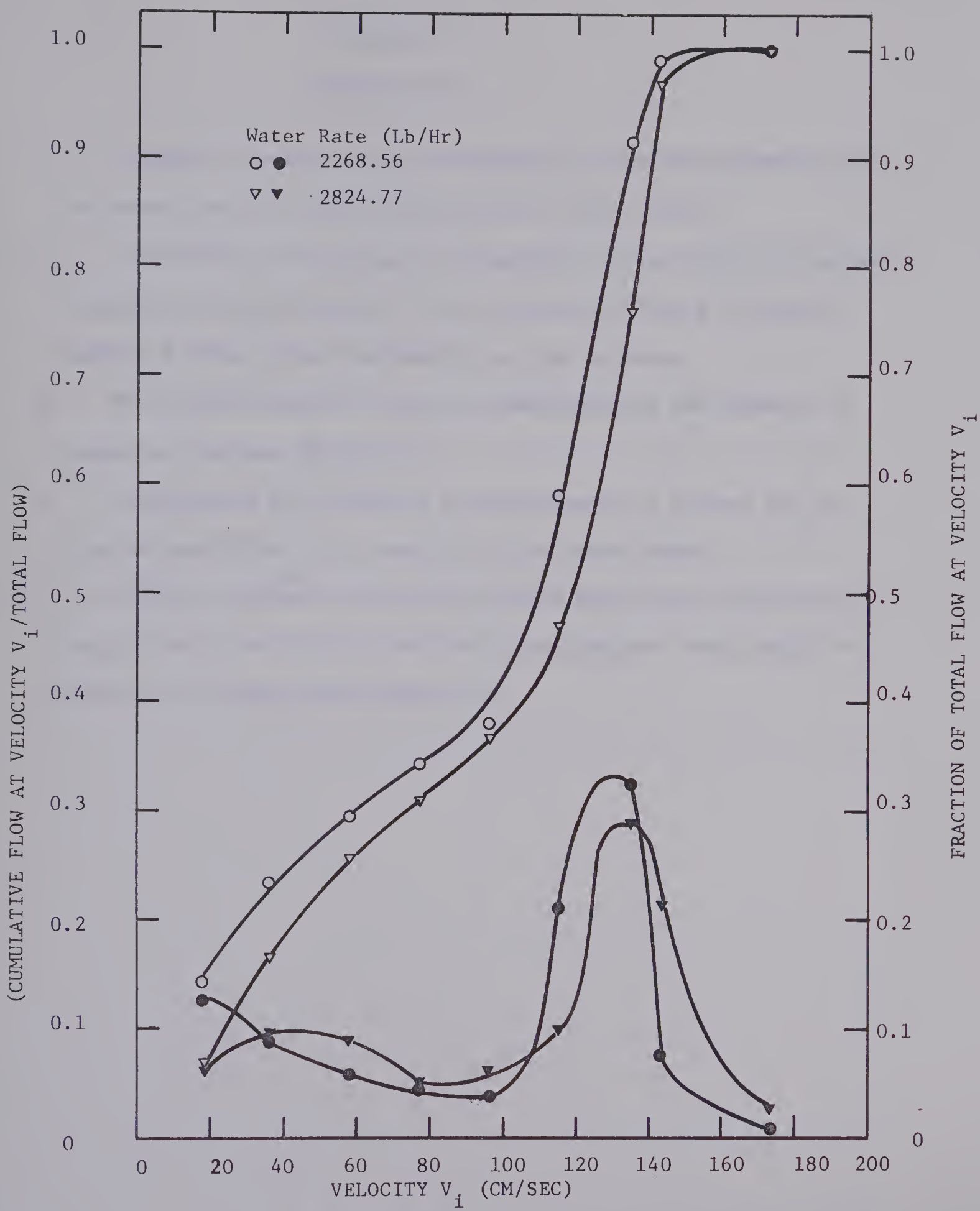


FIGURE 29. CUMULATIVE FLOW AND FRACTIONAL FLOW VERSUS VELOCITY AT AIR RATE OF 13.64 LB/HR FOR 1 3/4 INCH TUBE.

CHAPTER VI

CONCLUSIONS

1. Pressure fluctuations are independent of the tube diameter for the same no-slip or slug velocity in the tubes tested.
2. The density of the slugs is independent of the tube size and only dependent on slug velocity. Up to a no-slip velocity of approximately 3 ft/sec, the slug density is that of water.
3. The liquid fraction in slug is independent of the diameter of tubes for the same $(Q_L + Q_G)/Q_L$.
4. The measured slug velocity is approximately 1.2 times the no-slip or superficial slug velocity in the tubes tested.
5. The slug frequency versus slug Froude number plot (as defined by Gregory and Scott (25)) of the data taken compared fairly well with the data of Hubbard and Gregory-Scott.

CHAPTER VII

RECOMMENDATIONS

1. The effect of the inclination of tubes might be worth experimenting to simulate field pipeline operations.
2. For the one and three-quarter inch tube, data should be taken at a higher range of slug velocity to compare with the corresponding data for small diameter tubes.
3. Apart from water, different liquids e.g. sugar solutions, polymer solutions should be tried to study the effect of viscosity and density of the liquid.
4. Velocity distribution should be studied in detail for different tube sizes to establish accurate pressure drop predictions.
5. Film flow characteristics should be studied so as to predict holdup and film pressure drops.
6. Subatmospheric pressure fluctuations should be studied in detail.

NOMENCLATURE

A	tube cross sectional area
D	tube diameter
f	Fanning friction factor
g	gravitational acceleration, gravitational conversion factor
G	mass velocity of gas
L	length, mass velocity of liquid
N	number
P	pressure
Q	volumetric flow rate
R	holdup
R_e	Reynolds number
r	radius
v	velocity
W	mass flow rate
X	$\left[\left(\frac{\Delta P}{\Delta L} \right)_L / \left(\frac{\Delta P}{\Delta L} \right)_G \right]^{1/2}$ Lockhart-Martinelli parameter
Greek Letters	
Δ	difference
λ	$(\rho_G/0.075) (\rho_L/62.4)^{1/2}$ = parameter of Baker area plot for two-phase flow
μ	viscosity
π	3.14159 . . .
ρ	density
σ	surface tension

$$\phi \left[\left(\frac{\Delta P}{\Delta L} \right)_{TP} / \left(\frac{\Delta P}{\Delta L} \right)_L \right]^{1/2} \quad \text{Lockhart Martinelli parameter}$$

$$\psi \quad (73/\sigma) \left[\mu_L (62.3/\rho_L) \right]^{-1/3} = \text{parameter of Baker area plot}$$

for two-phase flow

Subscripts

C	constant
f	from friction considerations; fluctuations
G	gas
L	liquid
m	from momentum considerations
os	for one slug
s	slug flow
TP	two-phase
t	test
TOT	total

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APPENDIX -A

- (1) AUXILIARY DATA
- (2) EXPERIMENTAL DATA
- (3) CALCULATED DATA

A-1 AUXILIARY DATA

PIPELINE MATERIAL - CLEAR EXTRUDED LUCITE

TABLE A-1.1

TUBE DIAMETER	TEST SECTION LENGTH	ENTRANCE LENGTH
FEET	FEET	FEET
0.0417	30.17	6.08
0.0834	13.04	23.21
0.1468	21.08	15.17

PHYSICAL PROPERTIES OF FLUIDS

(AT 74°F, 14.7 PSIA)

	WATER	AIR
Viscosity	0.9358 cp	0.0175 cp
Density	62.20 lb/ft ³	0.0743 lb/ft ³
Surface Tension	72.30 dynes/cm	

A-2 EXPERIMENTAL DATA

FOR 1/2 INCH TUBE

TABLE A-2.1

RATES		FREQUENCY	% TIME-ON SLUG	FRACTION OF LIQUID IN SLUG	SLUG VEL	PRESSURES	
AIR	WATER					ΔP	ΔP_f
CC/SEC	CC/SEC	SEC ⁻¹			CM/SEC	CM HG	CM HG
72.0	32.5	1.20	26.2	0.82	80	2.2	0.20
	54.0	2.41	41.3	0.85	100	4.4	0.30
	73.5	3.17	54.2	0.84	120	6.4	0.42
	90.5	3.29	62.2	0.89	140	8.8	0.55
	103.5	3.54	68.7	0.88	152	10.7	0.65
154.0	32.5	1.06	17.4	0.77	162	3.2	0.75
	54.0	1.63	27.4	0.81	182	6.2	1.00
	73.5	2.15	35.1	0.81	206	8.4	1.12
	90.5	2.60	41.5	0.80	222	11.8	1.30
	103.5	3.05	49.4	0.83	232	14.2	1.50
218.0	32.5	0.83	13.9	0.73	222	4.8	1.30
	54.0	1.31	24.3	0.75	252	8.4	1.65
	73.5	1.78	30.6	0.76	265	12.0	2.10
	90.5	2.29	38.6	0.76	283	14.7	2.50
	103.5	2.50	43.9	0.75	304	17.8	2.70
285.0	32.5	0.71	10.9	0.72	304	5.4	2.40
	54.0	1.12	18.9	0.75	325	10.7	3.00
	73.5	1.55	25.5	0.77	345	14.2	3.20
	90.5	1.87	31.1	0.76	375	18.2	3.60
	103.5	2.25	38.3	0.77	385	21.3	3.80
357.0	32.5	0.59	9.4	0.68	385	5.4	3.80
	54.0	0.93	17.6	0.71	395	11.6	4.40
	73.5	1.41	23.5	0.73	417	16.8	4.40
	90.5	1.70	29.0	0.73	437	22.2	5.30
	103.5	2.02	34.5	0.75	458	25.7	6.50

EXPERIMENTAL DATA (CONTD.)

SEPARATOR BOX DATA FOR 1/2 INCH TUBE

AIR RATE CC/SEC	COMPARTMENT NUMBER	TOTAL LIQUID IN CC/MINUTE AT WATER RATES (CC/SEC) OF				
		32.5	54.0	73.5	90.5	103.5
72.0	2	73	25	0	0	0
	3	110	98	53	15	0
	4	108	120	122	104	48
	5	70	120	122	138	132
	6	750	156	175	136	177
	7	1030	885	240	200	167
	8	-	2060	3780	355	250
	9	-	-	-	4500	730
	10	-	-	-	310	5000
	11	-	-	-	-	160
154.0	2	124	52	17	0	0
	3	75	96	82	38	5
	4	75	94	120	110	70
	5	48	78	108	137	140
	6	45	88	110	143	160
	7	40	68	105	133	152
	8	55	71	90	115	146
	9		92	113	114	118
	10			113	152	160
	11				175	162
	12	1500	2690			205
	13			3670		
	14				4430	
	15					5120
	16					
	17					
	18					
218.0	2	84	60	12	0	0
	3	70	99	85	56	18
	4	50	86	115	106	87
	5	44	76	116	132	121
	6	44	79	104	132	160
	7	42	70	90	116	127
	8	35	70	97	122	135
	9	36	48	70	91	122
	10	107	71	74	98	123
	11	108	71	90	106	118
	12	345	112	81	124	128
	13	405		80	96	138
	14	310			132	120
	15	160				202

EXPERIMENTAL DATA (CONTD.)

SEPARATOR BOX DATA FOR 1/2 INCH TUBE

AIR RATE CC/SEC	COMPARTMENT NUMBER	TOTAL LIQUID IN CC/MINUTE AT WATER RATES (CC/SEC) OF				
		32.5	54.0	73.5	90.5	103.5
218.0 (CONTD.)	16	67	2530	↓	↓	↓
	17	10	↓	3360	↓	↓
	18			↓	4240	↓
	19				↓	4480
	20					↓
	21				↓	
	22					↓
	23					↓
	24					↓
285.0	2	172	67	28	0	0
	3	62	84	83	56	22
	4	50	76	112	112	72
	5	38	75	102	130	116
	6	38	75	100	138	162
	7	36	60	96	118	140
	8	35	59	92	113	140
	9	28	55	75	110	124
	10	27	47	65	95	107
	11	27	52	62	85	100
	12	27	50	72	95	102
	13	↓	38	55	76	100
	14		54	55	76	94
	15			47	70	85
	16				54	69
	17					77
	18					↓
	19	1400				↓
	20	↓	2485			↓
	21			3405		↓
	22				4210	↓
	23					4900
	24					↓
	25					
	26	↓	↓		↓	
	27			↓		
	28				↓	
	29					↓

EXPERIMENTAL DATA (CONTD.)

SEPARATOR BOX DATA FOR 1/2 INCH TUBE

AIR RATE CC/SEC	COMPARTMENT NUMBER	TOTAL LIQUID IN CC/MINUTE AT WATER RATES (CC/SEC) OF				
		32.5	54.0	73.5	90.5	103.5
357.0	2	116	58	19	0	0
	3	80	110	98	73	30
	4	64	92	122	118	85
	5	50	91	127	136	142
	6	47	78	118	138	162
	7	31	74	98	109	140
	8	42	60	90	98	118
	9	32	57	77	105	113
	10	30	58	69	96	100
	11	28	44	75	79	100
	12	26	44	62	78	87
	13	36	40	62	70	76
	14	35	42	63	74	96
	15	50	40	66	65	92
	16		47	56	69	70
	17		70	72	74	87
	18			74	62	64
	19				120	70
	20					124
	21					
	22					
	23					
	24	1325				
	25		2300			
	26			3200		
	27				3840	
	28					4360
	29					
	30					
	31					
	32					
	33					
	34					
	35					
	36					
	37					
	38					

EXPERIMENTAL DATA (CONTD.)

FOR 1 INCH TUBE

TABLE A-2.2

RATES		FREQUENCY	%TIME-ON SLUG	FRACTION OF LIQUID IN SLUG	SLUG VEL	PRESSURES	
AIR	WATER					ΔP	ΔP_f
CC/SEC	CC/SEC	SEC ⁻¹			CM/SEC	CM H ₂ O	CM H ₂ O
190	48	0.150	12.1	0.75	58	2.2	2.0
	143	0.583	32.2	0.82	68	6.0	3.0
	238	1.283	43.4	0.88	87	10.0	3.4
	334	1.833	64.5	0.91	97	15.0	3.5
	430	2.467	70.4	0.95	117	26.0	3.7
435	48	0.133	7.4	0.72	97	8.0	4.5
	143	0.550	18.5	0.75	127	13.0	6.0
	238	1.000	29.7	0.74	147	19.0	8.3
	334	1.550	42.0	0.82	166	25.0	10.3
	430	2.033	54.1	0.83	195	36.0	10.3
705	48	0.100	5.6	0.69	166	8.3	8.6
	143	0.433	14.6	0.71	195	13.0	10.8
	238	0.817	22.0	0.75	215	24.0	18.0
	334	1.367	31.0	0.77	245	34.0	17.5
	430	1.683	40.5	0.80	263	49.0	19.0
1040	48	0.100	3.6	0.63	245	10.7	15.5
	143	0.350	11.7	0.69	263	23.3	22.5
	238	0.650	17.7	0.70	314	42.0	22.5
	334	0.933	25.3	0.74	323	49.0	28.0
	430	1.450	31.9	0.72	362	68.0	36.4
1445	48	0.066	2.9	0.65	323	13.0	38.0
	143	0.333	10.6	0.68	362	28.0	41.8
	238	0.617	16.3	0.66	412	37.0	53.7
	334	0.867	22.5	0.71	440	52.0	57.2
	430	1.267	28.1	0.72	480	75.0	71.0

EXPERIMENTAL DATA (CONTD.)
SEPARATOR BOX DATA FOR 1 INCH TUBE

AIR RATE CC/SEC	COMPARTMENT NUMBER	TOTAL LIQUID IN CC/MINUTE AT WATER RATES (CC/SEC) OF				
		48	143	238	334	430
190	1	710	285	0	0	0
	2	607	1230	180	0	0
	3	670	665	1405	360	30
	4	820	4090	1170	1460	645
	5	80	1825	7140	3720	1215
	6	-	100	3545	11160	2755
	7	-	-	170	3985	13090
	8	-	-	-	165	7180
	9	-	-	-	-	280
435	1	635	300	15	0	0
	2	195	550	235	0	0
	3	190	700	745	170	0
	4	330	650	895	795	270
	5	305	445	1005	1255	895
	6	570	630	840	1385	1530
	7	335	3405	890	1025	1630
	8	330	1690	5055	1395	1430
	9	65	190	4410	5800	1710
	10	-	20	345	7520	5930
	11	-	-	20	470	10860
	12	-	-	-	50	1115
	13	-	-	-	-	155
705	1	570	305	0	0	0
	2	190	410	230	0	0
	3	155	475	620	195	0
	4	140	440	725	715	260
	5	95	385	685	880	705
	6	95	380	675	1010	1100
	7	185	265	555	920	1050
	8	155	265	485	855	1155
	9	405	510	475	670	985
	10	300	1455	670	765	1105
	11	355	1730	1735	870	1100
	12	225	955	3355	1995	965
	13	45	375	2035	4045	1430
	14	25	225	955	4130	5520
	15	-	48	440	2060	6290
	16	-	20	180	565	2100
	17	-	-	35	225	1150
	18	-	-	-	120	570
	19	-	-	-	-	170
	20	-	-	-	-	120
	21	-	-	-	-	100

EXPERIMENTAL DATA (CONTD.)

SEPARATOR BOX DATA FOR 1 INCH TUBE

AIR RATE CC/SEC	COMPARTMENT NUMBER	TOTAL LIQUID IN CC/MINUTE AT WATER RATES (CC/SEC) OF				
		48	143	238	334	430
1040	1	610	310	0	0	0
	2	200	435	185	0	0
	3	165	445	550	270	50
	4	125	415	690	565	360
	5	90	480	585	795	660
	6	75	390	615	875	935
	7	85	320	510	700	960
	8	82	250	460	700	920
	9	45	220	400	620	905
	10	44	230	380	585	975
	11	84	190	335	520	850
	12	260	220	330	525	740
	13	300	705	275	405	575
	14	220	1025	500	465	625
	15	160	1075	1340	810	810
	16	70	675	1455	1310	525
	17	-	635	2000	2715	1500
	18	-	385	1385	2575	2660
	19	-	175	715	1555	2590
	20	-	135	645	1390	3155
	21	-	160	465	880	2120
	22	-	-	180	425	1235
	23	-	-	225	355	1055
	24	-	-	110	220	490
	25	-	-	80	185	435
	26	-	-	-	130	240
	27	-	-	-	95	210
	28	-	-	-	125	195
	29	-	-	-	75	120
	30	-	-	-	-	60
	31	-	-	-	-	115
	32	-	-	-	-	90
1445	1	655	95	0	0	0
	2	425	420	150	0	0
	3	370	665	485	230	0
	4	240	705	835	490	315
	5	245	715	680	705	570
	6	275	580	715	905	895
	7	60	450	580	670	855
	8	55	360	540	640	820
	9	95	215	500	580	745
	10	45	215	425	610	755
	11	20	195	375	530	725
	12	15	160	335	485	720

EXPERIMENTAL DATA (CONTD.)

SEPARATOR BOX DATA FOR 1 INCH TUBE

AIR RATE	COMPARTMENT NUMBER	TOTAL LIQUID IN CC/MINUTE AT WATER RATES (CC/SEC) OF				
CC/SEC		48	143	238	334	430
1445	13	20	130	245	375	555
(CONTD.)	14	20	135	250	415	520
	15	15	155	310	440	600
	16	25	170	235	300	470
	17	5	575	325	365	580
	18	20	610	530	395	490
	19	-	340	575	470	335
	20	-	380	915	970	525
	21	-	400	900	1420	810
	22	-	260	815	1475	1395
	23	-	180	955	1485	1860
	24	-	105	570	1030	1655
	25	-	95	550	1105	1950
	26	-	50	345	730	1545
	27	-	30	250	520	950
	28	-	30	235	575	1170
	29	-	-	135	265	540
	30	-	-	90	195	435
	31	-	-	155	335	615
	32	-	-	105	235	455
	33	-	-	50	95	180
	34	-	-	25	55	95
	35	-	-	35	125	175
	36	-	-	35	100	175
	37	-	-	35	80	180
	38	-	-	140	410	1070

EXPERIMENTAL DATA (CONTD.)

FOR 1 3/4 INCH TUBE

TABLE A-2.3

RATES		FREQUENCY SEC ⁻¹	%TIME-ON SLUG	FRACTION OF LIQUID IN SLUG	SLUG VEL CM/SEC	PRESSURES	
AIR CC/SEC	WATER CC/SEC					ΔP CM H ₂ O	ΔP_f CM H ₂ O
435	143	0.0167	-	0.81	38.4	1.4	-
	215	0.1000	-	0.83	57.5	1.8	1.2
	287	0.1333	-	0.83	67.5	2.6	1.3
	358	0.1500	22.3	0.82	76.8	2.9	1.3
	430	0.1833	24.4	0.82	76.8	3.4	1.5
634	143	0.0167	-	0.75	76.8	1.4	1.1
	215	0.0833	-	0.75	76.8	2.1	1.4
	287	0.1000	15.5	0.76	76.8	2.6	1.5
	358	0.1333	17.6	0.80	87.0	3.2	1.6
	430	0.1667	21.8	0.81	87.0	3.5	1.6
860	143	0.0167	-	0.69	87.0	2.4	1.6
	215	0.0667	8.7	0.72	97.0	2.4	1.7
	287	0.0833	11.8	0.75	97.0	3.2	2.0
	358	0.1333	15.1	0.75	97.0	4.6	2.0
	430	0.1833	18.5	0.75	97.0	4.8	2.3
1137	143	0.0167	-	0.68	97.0	2.6	1.9
	215	0.0667	8.6	0.69	106.0	3.3	2.0
	287	0.0833	11.0	0.72	106.0	3.7	2.0
	358	0.1500	13.5	0.75	115.0	5.1	2.2
	430	0.2000	15.2	0.72	125.0	5.9	2.4
1445	143	0.0167	4.7	0.67	115.0	3.0	2.3
	215	0.0667	7.1	0.69	125.0	4.6	2.2
	287	0.0833	9.2	0.70	135.0	4.8	2.7
	358	0.1833	11.1	0.71	144.0	5.9	3.0
	430	0.2333	12.9	0.72	153.0	6.4	3.1

EXPERIMENTAL DATA (CONTD.)

SEPARATOR BOX DATA FOR 1 3/4 INCH TUBE

AIR RATE CC/SEC	COMPARTMENT NUMBER	TOTAL LIQUID IN CC/MINUTE AT WATER RATES (CC/SEC) OF				
		143	215	287	358	430
435	1	0	0	0	0	0
	2	180	70	0	0	0
	3	7720	8790	7980	7660	7485
	4	760	3845	4830	2735	2780
	5	-	1100	5590	10965	11760
	6	-	-	90	1060	2830
634	1	0	0	0	0	0
	2	2010	1380	450	55	60
	3	1085	6350	7790	8300	8110
	4	630	530	420	1585	2205
	5	4285	4525	8710	9180	7860
	6	-	110	1270	3035	7270
	7	-	-	-	85	180
860	1	0	0	0	0	0
	2	4480	2685	1990	870	500
	3	1790	3320	4340	5970	4740
	4	535	1085	1410	1735	3110
	5	1160	2110	3665	3020	3070
	6	800	2490	6420	9940	11280
	7	-	50	115	510	2480
	8	-	-	-	-	205
1137	1	0	110	55	0	0
	2	3065	3035	2475	2015	995
	3	2185	1250	3485	2830	3240
	4	335	985	1015	2020	2780
	5	530	510	990	1605	2390
	6	1155	3790	4350	3260	3020
	7	600	1740	4515	8025	8940
	8	-	110	650	960	5190
	9	-	-	-	-	390
1445	1	510	640	250	120	0
	2	2930	2340	2420	1400	1020
	3	870	1170	1760	2060	3600
	4	290	740	1120	1890	2140
	5	165	650	920	1140	2140
	6	520	970	710	1350	1720
	7	3130	3510	4000	2120	1530
	8	550	2780	6140	6180	3420
	9	-	40	1450	4520	6430
	10	-	-	160	600	1960
	11	-	-	-	-	400

A-3 CALCULATED DATA

FOR 1/2 INCH TUBE

TABLE A-3.1

AIR LB/HR	RATES WATER LB/HR	OVERALL PRESSURE DROP PSI	PRESSURE GRADIENT PSI/FT			
			DATA	SLUG THEORY -VERMEULEN	LOCKHART- MARTINELLI	KORDYBAN
0.68	256.89	0.424	0.0141	0.0185	0.0136	0.0098
	426.84	0.849	0.0282	0.0343	0.0272	0.0193
	580.97	1.235	0.0410	0.0467	0.0421	0.0297
	715.35	1.698	0.0564	0.0558	0.0567	0.0401
	818.10	2.065	0.0685	0.0640	0.0689	0.0490
1.45	256.89	0.617	0.0205	0.0346	0.0166	0.0152
	426.84	1.196	0.0397	0.0587	0.0326	0.0281
	580.97	1.621	0.0538	0.0815	0.0497	0.0415
	715.35	2.277	0.0757	0.1019	0.0665	0.0545
	818.10	2.740	0.0910	0.1196	0.0803	0.0653
2.06	256.89	0.926	0.0307	0.0412	0.0184	0.0189
	426.84	1.621	0.0538	0.0715	0.0356	0.0343
	580.97	2.316	0.0769	0.1010	0.0540	0.0500
	715.35	2.837	0.0943	0.1302	0.0718	0.0649
	818.10	3.435	0.1188	0.1483	0.0866	0.0771
2.69	256.89	1.042	0.0346	0.0478	0.0200	0.0226
	426.84	2.065	0.0685	0.0831	0.0382	0.1405
	580.97	2.740	0.0910	0.1187	0.0576	0.0584
	715.35	3.512	0.1167	0.1488	0.0765	0.0752
	818.10	4.110	0.1368	0.1773	0.0920	0.0888
3.37	256.89	1.042	0.0346	0.0507	0.0318	0.0264
	426.84	2.238	0.0743	0.0883	0.0562	0.0468
	580.97	3.242	0.1075	0.1309	0.0811	0.0670
	715.35	4.284	0.1424	0.1641	0.1046	0.0858
	818.10	4.960	0.1646	0.1941	0.1236	0.1009

CALCULATED DATA

FOR 1/2 INCH TUBE (CONTD.)

	RATES		PRESS	NO.OF	(PRESS	EFFECTIVE	NO-SLIP	MEASURED
	AIR	WATER	FLUCT.	SLUGS/	FUCT/FT)	DENSITY	VELOCITY	SLUG
	LB/HR	LB/HR	(PSI)	SECTION	^x (NO.OF	(GMS/C.C)	FT/SEC.	
					SLUGS/SECT)			VELOCITY
					(PSI/FT)			FT/SEC.
0.68	256.89	0.0387	13.72	0.0174	1.00	2.70	2.62	
	426.84	0.0580	21.94	0.0418	0.89	3.26	3.28	
	580.97	0.0812	23.77	0.0635	0.75	3.76	3.94	
	715.35	0.1064	21.94	0.0768	0.75	4.20	4.59	
	818.10	0.1257	21.03	0.0870	0.72	4.54	4.99	
1.45	256.89	0.1450	5.95	0.0284	0.72	4.82	5.31	
	426.84	0.1934	8.14	0.0518	0.71	5.38	5.97	
	580.97	0.2162	9.14	0.0651	0.66	5.88	6.76	
	715.35	0.2514	10.97	0.0904	0.63	6.32	7.28	
	818.10	0.2900	11.88	0.1134	0.62	6.66	7.61	
2.06	256.89	0.2514	3.38	0.0279	0.62	6.47	7.28	
	426.84	0.3191	4.75	0.0498	0.60	7.03	8.27	
	580.97	0.4062	6.13	0.0819	0.58	7.53	8.69	
	715.35	0.4836	7.41	0.1180	0.54	7.97	9.28	
	818.10	0.5222	7.50	0.1280	0.53	8.31	9.97	
2.69	256.89	0.4642	2.10	0.0321	0.57	8.21	9.97	
	426.84	0.5803	3.11	0.0594	0.53	8.76	10.66	
	580.97	0.6189	4.12	0.0839	0.51	9.27	11.32	
	715.35	0.6963	4.57	0.1046	0.47	9.71	12.30	
	818.10	0.7350	5.30	0.1282	0.44	10.04	12.63	
3.37	256.89	0.7350	1.37	0.0331	0.48	10.06	12.63	
	426.84	0.8511	2.20	0.0615	0.44	10.62	12.96	
	580.97	0.8511	3.11	0.0870	0.43	11.12	13.68	
	715.35	1.0251	3.57	0.1205	0.39	11.56	14.34	
	818.10	1.2572	4.02	0.1663	0.37	11.90	15.03	

CALCULATED DATA
FOR 1 INCH TUBE
TABLE A-3.2

AIR LB/HR	RATES WATER LB/HR	OVERALL PRESSURE GRADIENT (PSI/FT)				
		PRESSURE DROP PSI	DATA	SLUG THEORY -VERMEULEN	LOCKHART- MARTINELLI	KORDYBAN
1.79	379.41	0.0314	0.0024	0.0018	0.0012	0.0010
	1130.33	0.0853	0.0065	0.0067	0.0054	0.0040
	1881.24	0.1422	0.0109	0.0128	0.0111	0.0082
	2640.06	0.2138	0.0164	0.0186	0.0183	0.0137
	3398.88	0.3700	0.0284	0.0251	0.0266	0.0202
4.11	379.41	0.0755	0.0058	0.0034	0.0016	0.0017
	1130.33	0.1850	0.0142	0.0137	0.0065	0.0060
	1881.24	0.2705	0.0207	0.0253	0.0132	0.0115
	2640.06	0.3560	0.0273	0.0386	0.0214	0.0182
	3398.88	0.5130	0.0393	0.0516	0.0308	0.0259
6.66	379.41	0.1181	0.0091	0.0043	0.0029	0.0023
	1130.33	0.1850	0.0142	0.0175	0.0097	0.0080
	1881.24	0.3418	0.0262	0.0331	0.0183	0.0149
	2640.06	0.4840	0.0371	0.0529	0.0285	0.0228
	3398.88	0.6980	0.0535	0.0679	0.0399	0.0318
9.82	379.41	0.1522	0.0117	0.0057	0.0039	0.0031
	1130.33	0.3318	0.0254	0.0211	0.0124	0.0103
	1881.24	0.5985	0.0458	0.0395	0.0227	0.0187
	2640.06	0.6980	0.0535	0.0585	0.0346	0.0281
	3398.88	0.9685	0.0742	0.0853	0.0479	0.0386
13.64	379.41	0.1850	0.0142	0.0062	0.0052	0.0039
	1130.33	0.3980	0.0305	0.0261	0.0155	0.0129
	1881.24	0.5260	0.0404	0.0492	0.0276	0.0230
	2640.06	0.7400	0.0567	0.0722	0.0415	0.0342
	3398.88	1.0690	0.0818	0.1023	0.0568	0.0463

CALCULATED DATA

FOR 1 INCH TUBE (CONTD.)

RATES	PRESS	NO.OF	(PRESS	EFFECTIVE	NO-SLIP	MEASURED	
AIR	WATER	FLUCT.	SLUGS/	FUCT/FT)	DENSITY	VELOCITY	SLUG
LB/HR	LB/HR	(PSI)	SECTION	x(NO.OF	(GMS/C.C)	FT/SEC.	VELOCITY
				SLUGS/SECT)			FT/SEC.
				(PSI/FT)			
1.79	379.41	0.0285	1.27	0.0028	1.00	1.54	1.90
	1130.33	0.0427	3.54	0.0116	1.00	2.15	2.23
	1881.24	0.0484	6.05	0.0224	1.00	2.77	2.85
	2640.06	0.0491	7.06	0.0262	1.00	3.39	3.18
	3398.88	0.0530	8.03	0.0322	0.93	4.01	3.84
4.11	379.41	0.0640	0.56	0.0027	0.98	3.12	3.18
	1130.33	0.0854	1.92	0.0126	0.89	3.73	4.17
	1881.24	0.1181	3.00	0.0272	0.80	4.35	4.82
	2640.06	0.1465	4.07	0.0457	0.77	4.97	5.45
	3398.88	0.1465	4.74	0.0533	0.66	5.59	6.40
6.66	379.41	0.1224	0.27	0.0025	0.73	4.87	5.45
	1130.33	0.1537	1.03	0.0121	0.70	5.48	6.40
	1881.24	0.2561	1.75	0.0344	0.72	6.09	7.05
	2640.06	0.2490	2.66	0.0508	0.67	6.71	8.04
	3398.88	0.2703	2.96	0.0613	0.64	7.33	8.63
9.82	379.41	0.2205	0.19	0.0032	0.62	7.03	8.04
	1130.33	0.3201	0.60	0.0147	0.65	7.64	8.63
	1881.24	0.3201	1.03	0.0253	0.60	8.26	10.30
	2640.06	0.3983	1.37	0.0418	0.59	8.88	10.60
	3398.88	0.5179	1.99	0.0790	0.54	9.50	11.88
13.64	379.41	0.5406	0.09	0.0037	0.54	9.65	10.60
	1130.33	0.5947	0.42	0.0192	0.48	10.26	11.88
	1881.24	0.7640	0.74	0.0433	0.47	10.87	13.52
	2640.06	0.8138	0.98	0.0611	0.43	11.49	14.44
	3398.88	1.0101	1.36	0.1053	0.40	12.11	15.75

CALCULATED DATA
FOR 1 3/4 INCH TUBE
TABLE A-3.3

RATES		OVERALL PRESSURE DROP PSI	PRESSURE GRADIENT (PSI/FT)			
AIR	WATER		DATA	SLUG	LOCKHART-	KORDYBAN
LB/HR	LB/HR			THEORY	MARTINELLI	
				-VERMEULEN		
4.11	1130.33	0.020	0.00094	0.00051	0.00048	0.00039
	1699.44	0.026	0.00121	0.00097	0.00083	0.00066
	2268.56	0.037	0.00175	0.00152	0.00125	0.00098
	2829.77	0.041	0.00195	0.00191	0.00171	0.00132
	3398.88	0.048	0.00229	0.00239	0.00223	0.00171
5.99	1130.33	0.020	0.00094	0.00065	0.00053	0.00049
	1699.44	0.030	0.00141	0.00139	0.00091	0.00081
	2268.56	0.037	0.00175	0.00186	0.00136	0.00117
	2829.77	0.046	0.00215	0.00246	0.00185	0.00157
	3398.88	0.050	0.00236	0.00308	0.00240	0.00200
8.12	1130.33	0.034	0.00161	0.00081	0.00057	0.00060
	1699.44	0.034	0.00161	0.00177	0.00099	0.00097
	2268.56	0.046	0.00215	0.00222	0.00146	0.00138
	2829.77	0.065	0.00310	0.00311	0.00198	0.00183
	3398.88	0.068	0.00323	0.00461	0.00256	0.00231
10.74	1130.33	0.037	0.00175	0.00098	0.00087	0.00072
	1699.44	0.047	0.00222	0.00197	0.00140	0.00116
	2268.56	0.053	0.00249	0.00267	0.00199	0.00163
	2829.77	0.073	0.00344	0.00394	0.00262	0.00213
	3398.88	0.084	0.00398	0.00561	0.00332	0.00268
13.64	1130.33	0.043	0.00202	0.00115	0.00102	0.00085
	1699.44	0.065	0.00310	0.00213	0.00162	0.00135
	2268.56	0.068	0.00323	0.00316	0.00228	0.00189
	2829.77	0.084	0.00398	0.00537	0.00299	0.00245
	3398.88	0.091	0.00431	0.00650	0.00376	0.00307

CALCULATED DATA

FOR 1 3/4 INCH TUBE (CONTD.)

RATES AIR LB/HR	WATER LB/HR	PRESS FLUCT. (PSI)	NO.OF SLUGS/ SECTION	(PRESS FUCT/FT x(NO.OF SLUGS/SECT) (PSI/FT)	EFFECTIVE DENSITY (GMS/C.C)	NO-SLIP VELOCITY FT/SEC.	MEASURED SLUG VELOCITY FT/SEC.
4.11	1130.33	-	0.29	-	-	1.21	1.26
	1699.44	0.0171	1.04	0.00084	-	1.36	1.89
	2268.56	0.0185	1.87	0.00164	-	1.51	2.21
	2829.77	0.0185	1.91	0.00168	0.99	1.65	2.52
	3398.88	0.0213	2.14	0.00217	1.01	1.80	2.52
5.99	1130.33	0.0156	0.22	0.00016	-	1.62	2.52
	1699.44	0.0199	0.99	0.00094	-	1.77	2.52
	2268.56	0.0213	1.10	0.00111	1.00	1.92	2.52
	2829.77	0.0228	1.36	0.00147	0.98	2.07	2.85
	3398.88	0.0228	1.58	0.00171	0.99	2.22	2.85
8.12	1130.33	0.0228	0.17	0.00018	-	2.09	2.85
	1699.44	0.0242	0.78	0.00090	0.99	2.24	3.18
	2268.56	0.0285	0.73	0.00099	1.00	2.39	3.18
	2829.77	0.0285	1.11	0.00149	0.99	2.54	3.18
	3398.88	0.0327	1.96	0.00304	1.01	2.69	3.18
10.74	1130.33	0.0270	0.13	0.00017	-	2.67	3.18
	1699.44	0.0285	0.50	0.00067	0.99	2.82	3.48
	2268.56	0.0285	0.59	0.00080	0.97	2.97	3.48
	2829.77	0.0313	1.01	0.00164	0.92	3.12	3.77
	3398.88	0.0341	1.61	0.00261	0.93	3.27	4.10
13.64	1130.33	0.0327	0.11	0.00016	0.93	3.31	3.77
	1699.44	0.0313	0.30	0.00045	0.90	3.46	4.10
	2268.56	0.0384	0.49	0.00089	0.89	3.61	4.43
	2829.77	0.0427	1.12	0.00227	0.88	3.76	4.72
	3398.88	0.0441	1.26	0.00263	0.87	3.91	5.02

B- SAMPLE CALCULATIONS

PROBLEM: To predict the two-phase pressure drop in 1.0 inch tube.

Air Rate = 1040 cc/sec.

Water Rate = 430 cc/sec.

PHYSICAL PROPERTIES
(at 74°F. and 14.7 PSIA)

	Viscosity (Centipoise)	Density ₃ (lb/ft ³)
Air	0.0175	0.0743
Water	0.9358	62.20

1. Solution by Lockhart-Martinelli Correlation
Rates converted to lbs/hr.

$$\begin{aligned}
 W_L &= Q_L (60.0)(0.002118)\rho_L \\
 &= (430)(60.0)(0.002118)(62.2) \\
 &= 3398.88 \text{ lb/hr}
 \end{aligned}$$

$$\begin{aligned}
 W_G &= Q_G (60.0)(0.002118)\rho_G \\
 &= (1040)(60.0)(0.002118)(0.0743) \\
 &= 9.82 \text{ lb/hr}
 \end{aligned}$$

Calculation of the Reynolds numbers for the gas and liquid phases; assuming each phase to flow alone.

$$\begin{aligned}
 R_{eL} &= (4.0)W_L / \mu_L (2.4191)\pi D. \\
 &= (4.0)(3398.88) / (0.9358)(2.4191)(\pi)(0.0834) \\
 &= 22,900
 \end{aligned}$$

$$\begin{aligned}
 R_{eG} &= (4.0)W_G / \mu_G (2.4191)\pi D \\
 &= (4.0)(9.82) / (0.0175)(2.4191)(\pi)(0.0834) \\
 &= 3,540
 \end{aligned}$$

Since the Reynolds numbers are above 2000, both phases are in turbulent flow.

The Blasius numerators and exponents for this case are:

liquid	gas
$n = 0.2$	$m = 0.2$
$C_L = 0.046$	$C_G = 0.046$
$f_L = 0.046 / R_{eL}^{0.2}$	
$= 0.046 / (22.900)^{0.2}$	
$= 0.046 / 7.447$	
$= 0.0062$	

Pressure drop in the liquid phase.

$$\begin{aligned}
 \Delta P_L &= (32.0) f_L W_L^2 / \rho_L D^5 \pi^2 g_c (3600.0)^2 (144.0) \\
 &= (32.0) (0.0062) (3398.88)^2 / (62.2) (0.0834)^5 \\
 &\quad \pi^2 (32.17) (3600.0)^2 (144.0) \\
 &= 0.0156 \text{ psi/ft.}
 \end{aligned}$$

Calculation of Parameter $X = \left\{ \left(\frac{\Delta P}{\Delta L} \right)_L / \left(\frac{\Delta P}{\Delta L} \right)_G \right\}^{0.5}$

For turbulent-turbulent flow:

$$\begin{aligned}
 X^2 &= \left(\frac{W_L}{W_G} \right)^{1.8} \frac{\rho_G}{\rho_L} \left(\frac{\mu_L}{\mu_G} \right)^{0.2} \\
 &= \left(\frac{3398.88}{9.82} \right)^{1.8} \left(\frac{0.0743}{62.2} \right) \left(\frac{0.9358}{0.0175} \right)^{0.2}
 \end{aligned}$$

$$X^2 = 99.7$$

$$X = 9.98$$

From Lockhart-Martinelli's Coordinates of ϕ and R_L verses parameter X .

$$\phi_L = \left\{ \left(\frac{\Delta P}{\Delta P} \right)_{TP} / \left(\frac{\Delta P}{\Delta L} \right)_L \right\}^{0.5} = 1.75$$

$$R_L = \text{liquid holdup} = 0.536$$

Therefore,

$$\begin{aligned}\Delta P_{TP} &= \phi_L^2 \Delta P_L \\ &= (1.75)^2 (0.0156) \\ &= 0.0479 \text{ psi/ft}\end{aligned}$$

2. Kordyban Correlation

Calculation of the Kordyban function

$$\begin{aligned}\text{KORFUN} &= 1.0 + (\rho_L / \rho_G)(W_G / W_L) \\ &= 1.0 + (62.2 / 0.0743)(9.82 / 3398.88) \\ &= 3.422\end{aligned}$$

Calculation of single phase pressure drop in liquid. Kordyban assumed only turbulent flow in smooth tubes with Reynolds number less than 100,000

Therefore,

$$f_s = (0.079)(R_e^{0.25}).$$

The Blasius equation used by Lockhart-Martinelli was assumed to be more applicable for extending Kordyban's correlation range.

Therefore, from Lockhart-Martinelli calculation:

$$P_L = 0.0156 \text{ psi/ft}$$

Now,

$$\begin{aligned}\Delta P_{TP} &= \Delta P_L (\text{KORFUN})^{0.75} \\ &= (0.0156)(3.422)^{0.75} \\ &= 0.0386 \text{ psi/ft}\end{aligned}$$

3. Slug Model Theory of Vermeulen

Given: Slug frequency = 1.45/sec.

Calculation of fraction of liquid in total flow:

$$\begin{aligned}\text{FRACTL} &= Q_L / (Q_L + Q_G) \\ &= 430 / (430 + 1040) \\ &= 0.292\end{aligned}$$

Calculation of no-slip velocity

$$\begin{aligned} V &= (Q_L + Q_G) / (\pi D^2/4) \\ &= (4.0)(1470)(3.5314 \times 10^{-5}) / (\pi)(0.0834)^2 \\ &= 9.50 \text{ ft/sec} \end{aligned}$$

Fluid Reynolds number, assuming fluid density and viscosity to be that of the liquid

$$\begin{aligned} R_{es} &= DV\rho_L/\mu_L \\ &= (0.0834)(9.50)(62.2)/(0.9358)(6.72 \times 10^{-4}) \\ &= 78,356 \end{aligned}$$

Friction factor by Blasius equation

$$\begin{aligned} f_s &= 0.0791/R_e^{0.25} \\ &= (0.0791)/(78356)^{0.25} \\ &= 0.00473 \end{aligned}$$

Frictional pressure drop of the slugs

$$\begin{aligned} \Delta P_s &= ((2.0)f\rho_L V^2/g_c D(144.0)) ((Q_L/(Q_L+Q_G))) \\ &= ((2.0)(0.00473)(62.2)(9.50)^2/(32.17)(0.0834)(144.0)) (0.292) \\ &= 0.0401 \text{ psi/ft} \end{aligned}$$

Calculation of momentum loss term

Number of slugs in the test section,

$$\begin{aligned} N_s &= (\text{slug frequency}/V) L_t \\ &= (1.45/9.50)(13.042) \\ &= 1.99 \text{ slugs/section} \end{aligned}$$

Momentum loss term

$$\begin{aligned} \Delta P_m &= (N_s L (R_L - \text{FRACTL}) V^2/g_c (144.0) L_t) \\ &= (1.45)(62.2)(0.536-0.292)(9.5)^2/(32.17)(144.0)(13.042) \\ &= 0.0452 \text{ psi/ft} \end{aligned}$$

Therefore, the total pressure drop by the slug flow model of Vermeulen is

$$\begin{aligned} \Delta P_{TOT} &= \Delta P_s + \Delta P_m \\ &= 0.0401 + 0.0452 \\ &= 0.0853 \text{ psi/ft} \end{aligned}$$

Actual pressure drop was 0.0742 psi/ft

Comparison

Relationship	ΔP_{TOT}	Deviation from actual
Lockhart-Martinelli	0.0479	-35.4%
Kordyban	0.0386	-48.0%
Slug model of Vermeulen	0.0853	+15.0%
Actual	0.0742	-

APPENDIX C

CALCULATION OF VELOCITY FROM COMPARTMENT NUMBER OF
SEPARATOR BOX

Use is made of the free-falling trajectory of liquid emerging from the tube exit, to establish the velocity from the compartment, of the separator box, into which it falls. Different equations have been derived for different tube sizes as follows:

C-1. For one-half inch diameter tube

Using the equation of trajectory,

$$x = Vt$$

$$y = \frac{1}{2} gt^2$$

$$V = x\sqrt{g/2y}$$

In this case, $y = 7 \text{ inches} = 7/12 \text{ ft.}$

$$g = 32.2 \text{ ft/sec}^2$$

$$\text{Therefore, } V = x\sqrt{(32.2)/(2)(7/12)}$$

$$V = 5.35x \quad ; \quad x = \text{ft, } V = \text{ft/sec}$$

$$V/30.48 \text{ (cm/sec)} = 5.35x/12 \text{ (in)}$$

$$V = 13.6x \quad ; \quad x = \text{in, } V = \text{cm/sec}$$

$$V = (13.6)(1.5)(n-2) \quad ; \quad V = \text{cm/sec, } n = \text{compartment number}$$

$$V = 20.4(n-2) \quad \quad \quad (\text{C-1.1})$$

This is the equation for establishing velocity from the compartment number for one-half inch tube.

C-2. For one inch diameter tube

In this case, $y = 7 \frac{1}{4} \text{ inches}$

$$= 29/48 \text{ ft.}$$

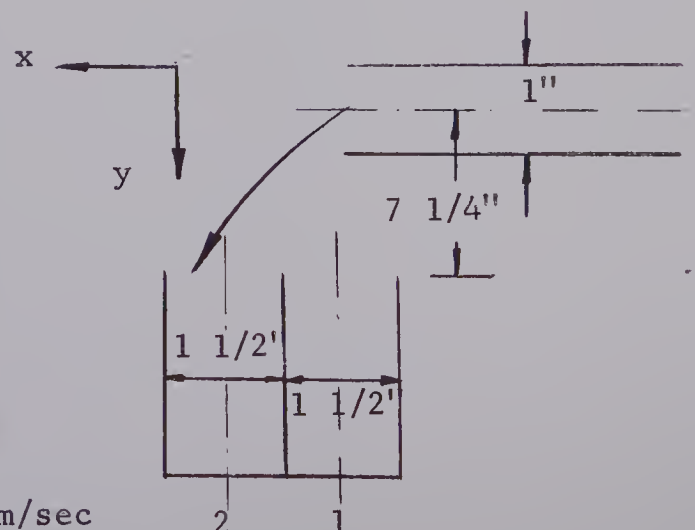
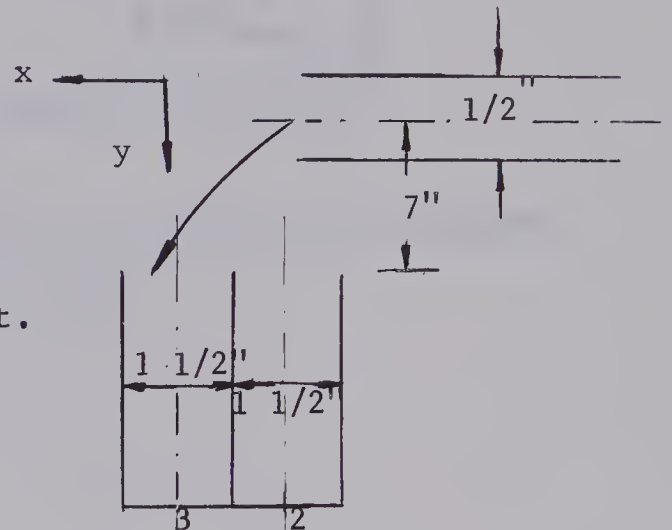
$$V = 5.16x \quad ; \quad x = \text{ft, } V = \text{ft/sec}$$

$$\text{Also, } V = 13.10x \quad ; \quad x = \text{in.,}$$

$$V = \text{cm/sec}$$

$$V = (13.10)(1.5)(n-1) \quad ; \quad V = \text{cm/sec}$$

$n = \text{compartment number}$



$$V = 19.65(n-1) \quad (C-2.1)$$

This is the equation for establishing velocity from the compartment number for one inch tube.

C-3. For one and three-quarter inch diameter tube

In this case, $y = 7 \frac{5}{8}$ in.

$$= 61/96 \text{ ft}$$

$$V = 5.034x ; \quad x = \text{ft}, \quad V = \text{ft/sec}$$

$$\text{Also, } V = 12.8x ; \quad x = \text{in.},$$

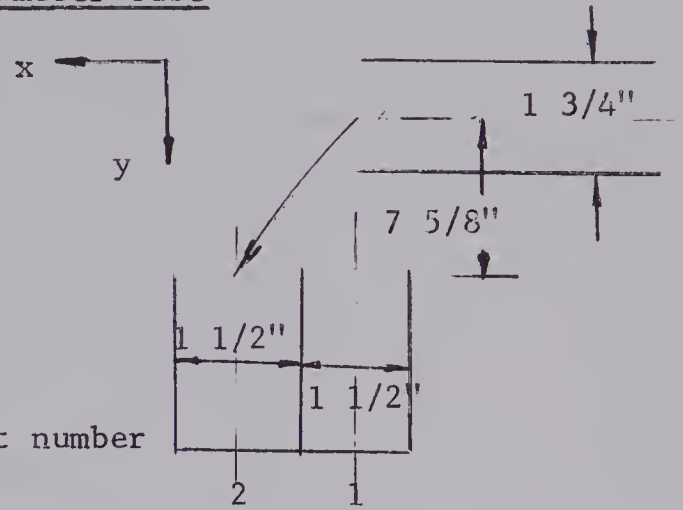
$$V = \text{cm/sec}$$

$$\text{And, } V = (12.8)(1.5)(n-1) ;$$

$$V = \text{cm/sec}, \quad n = \text{compartment number}$$

$$V = 19.2(n-1) \quad (C-3.1)$$

Equation (C-3.1) represents calculation of velocity from compartment number for one and three-quarter inch tube.



APPENDIX-D
CALIBRATIONS

- (1) ROTAMETERS
- (2) MAGNETIC FLOW TRANSMITTER
- (3) PRESSURE TRANSDUCERS

D-1 ROTAMETER CALIBRATION

Calibration of the water Rotameters was carried out by measuring the flow into a graduated cylinder over specific time intervals. The flow rates in cubic centimeters/minute were converted to cubic centimeters/second and shown in Table D-1.1 against scale readings.

For air Rotameters, calibration curves of Vermeulen (42) were used. The flow rates in cubic feet per minute at 80°F, 14.65 psia were converted to cubic centimeter per minute at 74°F, 14.7 psia versus the scale reading. The results are shown in Table D-1.2.

A representative plot for one of the calibrations for water Rotameter is shown in Figure 30.

TABLE D-1.1
ROTAMETER CALIBRATION DATA WITH
WATER AT 74°F

Brooks Rotameter #6709-31235
Model 6-1100 GTM, Tube R-6L-600 G 1, Float 316 SS (RS Spool)

Scale Reading	cc/sec
10	2.18
20	3.30
30	4.93
40	6.05
50	7.33
60	8.36
70	9.54
80	10.80
90	11.95
100	13.15

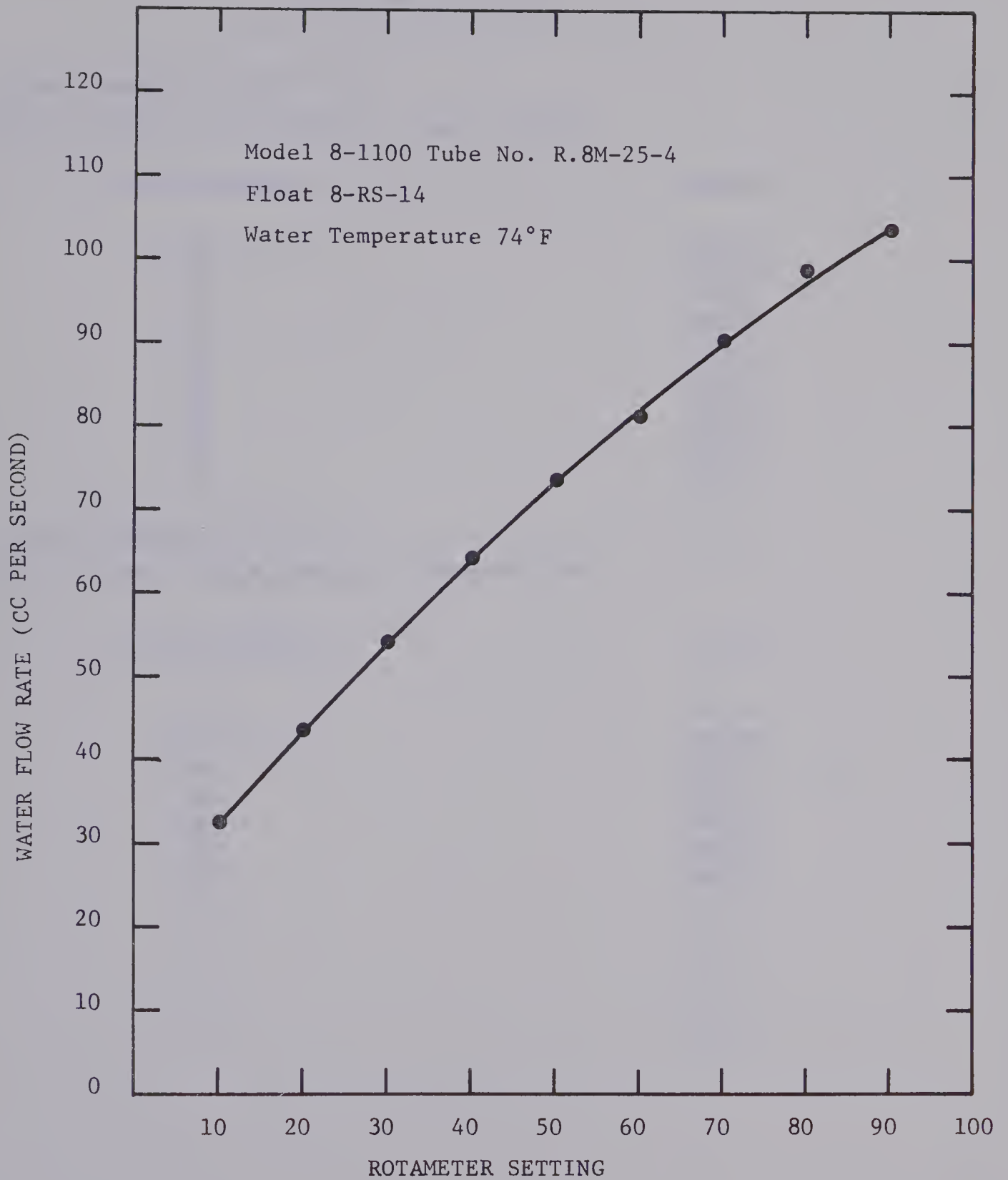


FIGURE 30 CALIBRATION OF BROOKS ROTAMETER # 624-675-1

TABLE D-1.1 (CONTD.)

Brooks Rotameter #624-675-1

Model 8-1100, Tube R-8M-25-4, Float 8-RS-14

Scale Reading	cc/sec
10	32.6
20	43.5
30	54.0
40	64.0
50	73.5
60	81.0
70	90.5
80	98.8
90	103.4

Brooks Rotameter #624-675-1

Model 8-1100, Tube R-8M-25-4, Float 8-LJ-48

Scale Reading	cc/sec
10	40.8
21	79.0
30	109.5
40	136.0
50	170.0
60	200.0
71	228.0

TABLE D-1.2

ROTAMETER CALIBRATION DATA WITH
AIR AT 74°F

Brooks Rotameter #6709-31236

Model 6-1100 GTM, Tube R-6L-600-G 1, Float 316 ss(RS Spool)

Scale Reading	cc/sec (74°F, 14.7 psia)
10	73.0
20	117.5
30	150.0
40	186.0
50	217.5
60	250.0
70	285.0
80	315.0
90	362.5

Brooks Rotameter #6709-2288-1

Model 8-1100, Tube R-8M-25-4, Float 8 R V3

Scale Reading	cc/sec (74°F, 14.7 psia)
10	185.0
20	305.0
30	434.0
40	555.0
50	705.0
60	860.0
70	1035.0
80	1240.0
90	1440.0

D-2 MAGNETIC FLOW TRANSMITTER CALIBRATION

The Foxboro Magnetic Flow Transmitter, Type 1895-SB TS-BA, # M-159992 B was calibrated by measuring the flows out of buckets over specific time intervals. The flows in cubic centimeter per minute were converted to cubic centimeter per second versus the readings on the Honeywell Electronic 16 Printer #5730268001. The resultant data are shown in Table D-2.1 and are plotted in Figure 31.

TABLE D-2.1

MAGNETIC FLOW TRANSMITTER CALIBRATION DATA WITH
WATER AT 74°F

Foxboro Magnetic Flow Transmitter # M-159992 B

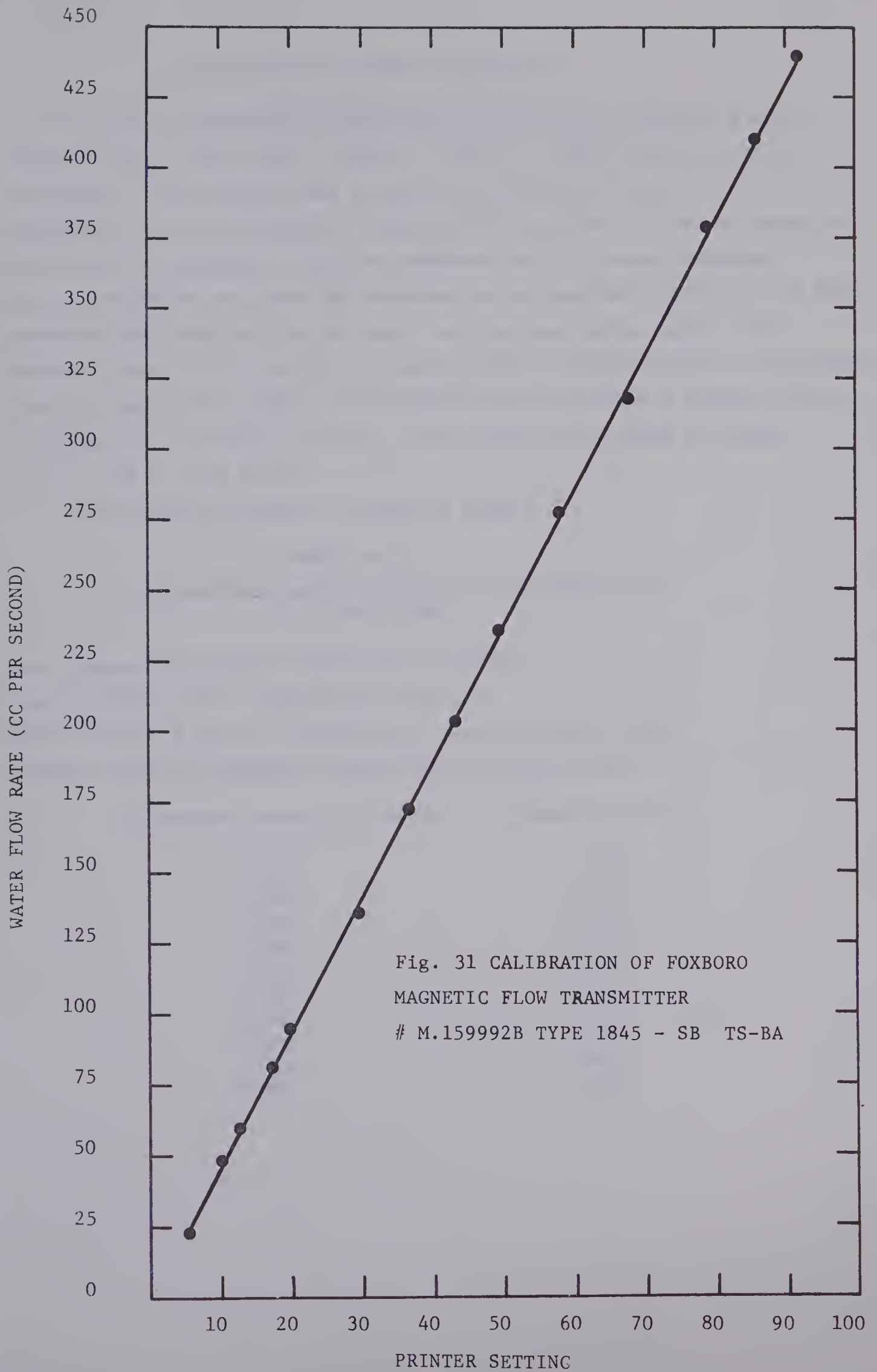
Type 1895-SB TS-BA, Size: 1/2 inch

Foxboro Magnetic Flow to Current Converter, Model-696

Honeywell Electronic 16 Printer #5730268001

Model No. Y 16305866 104-106-0000-01002

Printer Reading	cc/sec
5.0	23.5
10.0	48.8
12.5	58.9
17.0	82.3
20.0	94.5
29.5	142.3
36.5	172.5
43.0	203.0
49.0	235.8
58.0	277.2
68.0	318.0
79.0	377.8
85.5	409.8
91.5	440.3



D-3 PRESSURE TRANSDUCER CALIBRATION

The three Pace Wiancko (Whittaker Corporation) Transducers Model P7D⁺-100 PSID, P7D⁺-5 PSID, #23874, #23875, #23882 were calibrated as follows. The pressure was applied to the positive side of the transducer from the laboratory compressed air system, with the negative side open to atmosphere, and was measured with a U-tube manometer. Mercury manometer was used for calibration of one-half inch tube and water manometer was used for that of one, and one and three-quarter inch diameter tubes. This was done to get accurate calibration for low pressure drops in bigger tube sizes. The output was recorded on a Beckman (Type RS Dynograph) stripchart recorder. The results are shown in Tables D-3.1, D-3.2, and D-3.3.

A representative plot is shown in Figure 32.

TABLE D-3.1
CALIBRATION DATA FOR PRESSURE TRANSDUCERS FOR
1/2 INCH TUBE

Pace Wiancko (Whittaker Corporation) #23882
Model P7D⁺-100 PSID Diaphragm: 10 PSI
Hewlett-Packard Carrier Preamplifier Setting: Atten = 200
Beckman Type RS Dynograph Preamplifier Setting: Atten = 1

Dynograph Recording (volts)	Pressure (cm Hg)
0.125	2.3
0.350	6.0
0.600	10.0
0.800	14.0
0.930	16.8
1.125	19.8
1.300	23.0
1.525	27.0
1.750	31.2
1.900	33.6
2.000	35.6

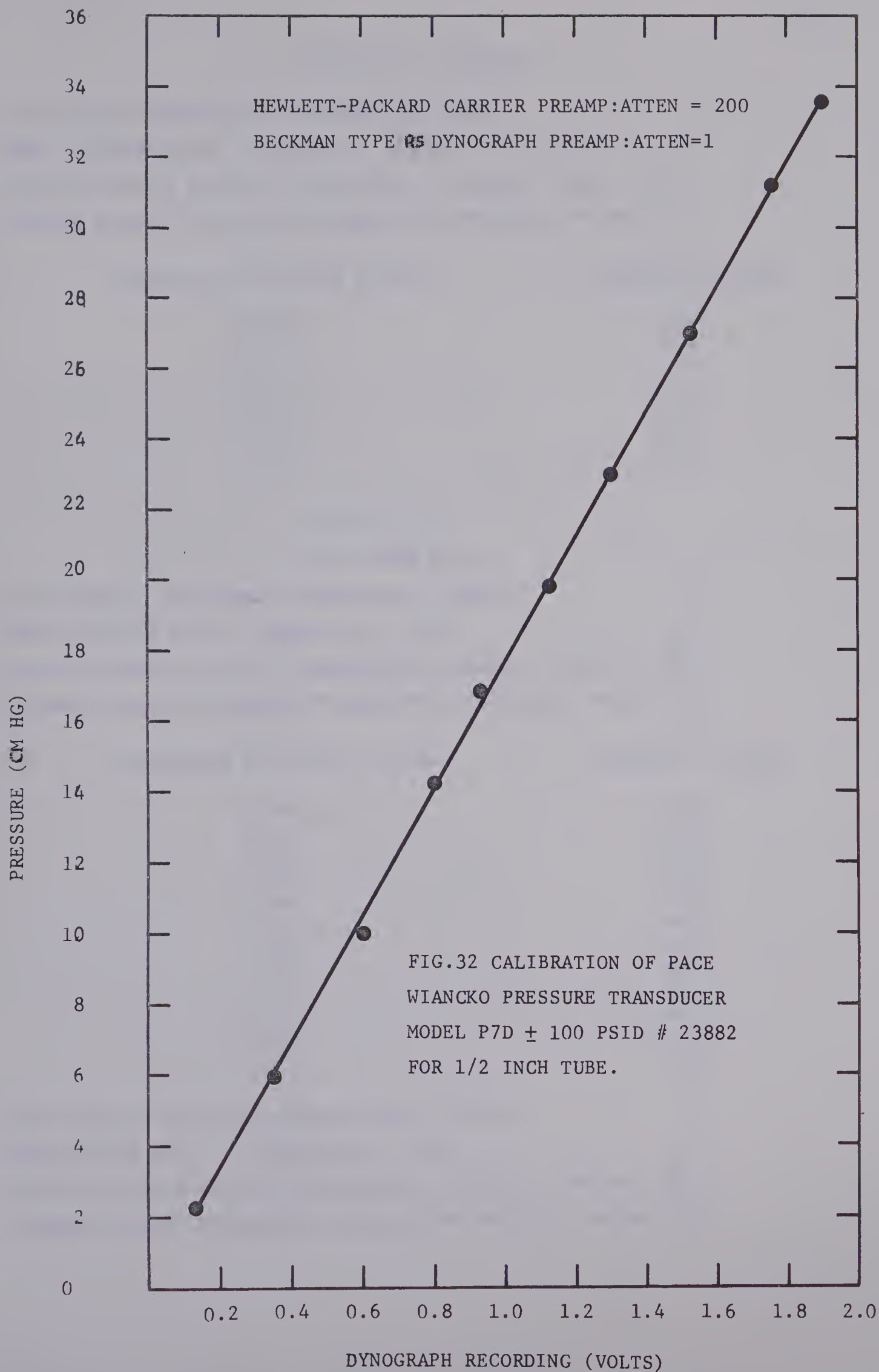


TABLE D-3.1 (CONTD.)

Pace Wiancko (Whittaker Corporation) #23874

Model P7D⁺-100 PSID Diaphragm: 5 PSI

Hewlett-Packard Carrier Preamplifier Setting: Atten = 50

Beckman Type RS Dynograph Preamplifier Setting: Atten = 1

Dynograph Recording (volts)	Pressure (cm Hg)
0.20	3.0
0.37	5.8
0.60	8.9
0.75	11.1
0.90	12.8
1.10	16.2
1.33	19.6

TABLE D-3.2

FOR 1-INCH TUBE

Pace Wiancko (Whittaker Corporation) #23874

Model P7D⁺-100 PSID Diaphragm: 1 PSI

Hewlett-Packard Carrier Preamplifier Setting: Atten = 100

Beckman Type RS Dynograph Preamplifier Setting: Atten = 1

Dynograph Recording (volts)	Pressure (cm H ₂ O)
0.40	15.0
0.55	20.0
0.70	25.0
0.85	30.0
1.00	35.0
1.15	40.0
1.30	45.0
1.45	50.0
1.60	55.0
1.75	60.0
2.03	70.0
2.13	75.0

Pace Wiancko (Whittaker Corporation) #23875

Model P7D⁺-5 PSID Diaphragm: 5 PSI

Hewlett-Packard Carrier Preamplifier Setting: Atten = 50

Beckman Type RS Dynograph Preamplifier Setting: Atten = 1

TABLE D-3.2 (CONTD.)

Dynograph Recording (volts)	Pressure (cm H ₂ O)
0.20	20.0
0.35	30.0
0.53	46.0
0.68	60.0
0.95	82.0
1.16	101.0
1.40	122.0
1.64	140.0
1.86	158.0

Pace Wiancko (Whittaker Corporation) #23874

Model P7D⁺-100 PSID Diaphragm: 2.5 PSI

Hewlett-Packard Carrier Preamplifier Setting: Atten = 50

Beckman Type RS Dynograph Preamplifier Setting: Atten = 1

Dynograph Recording (volts)	Pressure (cm H ₂ O)
0.42	20.0
0.64	30.0
0.83	40.0
1.25	60.0
1.50	70.0
1.74	80.0
1.90	90.0
2.15	100.0

TABLE D-3.3

FOR 1 3/4 INCH TUBE

Pace Wiancko (Whittaker Corporation) #23875

Model P7D⁺-5 PSID Diaphragm: 5 PSI

Hewlett-Packard Carrier Preamplifier Setting: Atten = 20

Beckman Type RS Dynograph Preamplifier Setting: Atten = 1

Dynograph Recording (volts)	Pressure (cm H ₂ O)
0.30	4.0
0.45	6.0
0.60	8.0
0.75	10.0
0.90	12.0
1.05	14.0
1.20	16.0

TABLE D-3.3 (CONTD.)

Pace Wiancko (Whittaker Corporation) #23874

Model P7D⁺-100 PSID Diaphragm: 1 PSI

Hewlett-Packard Carrier Preamplifier Setting: Atten = 50

Beckman Type RS Dynograph Preamplifier Setting: Atten = 1

Dynograph Recording (volts)	Pressure (cm H ₂ O)
0.55	4.0
0.80	6.0
1.07	8.0
1.35	10.0
1.64	12.0
1.95	14.0
2.20	16.0

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